NYS Statewide Language RBE-RN at the Metropolitan Center for Research and Equity at NYU

TEACHING MATH TO ELLS: RESOURCES AND STRATEGIES FOR TEACHERS

Richmond Hill H.S.

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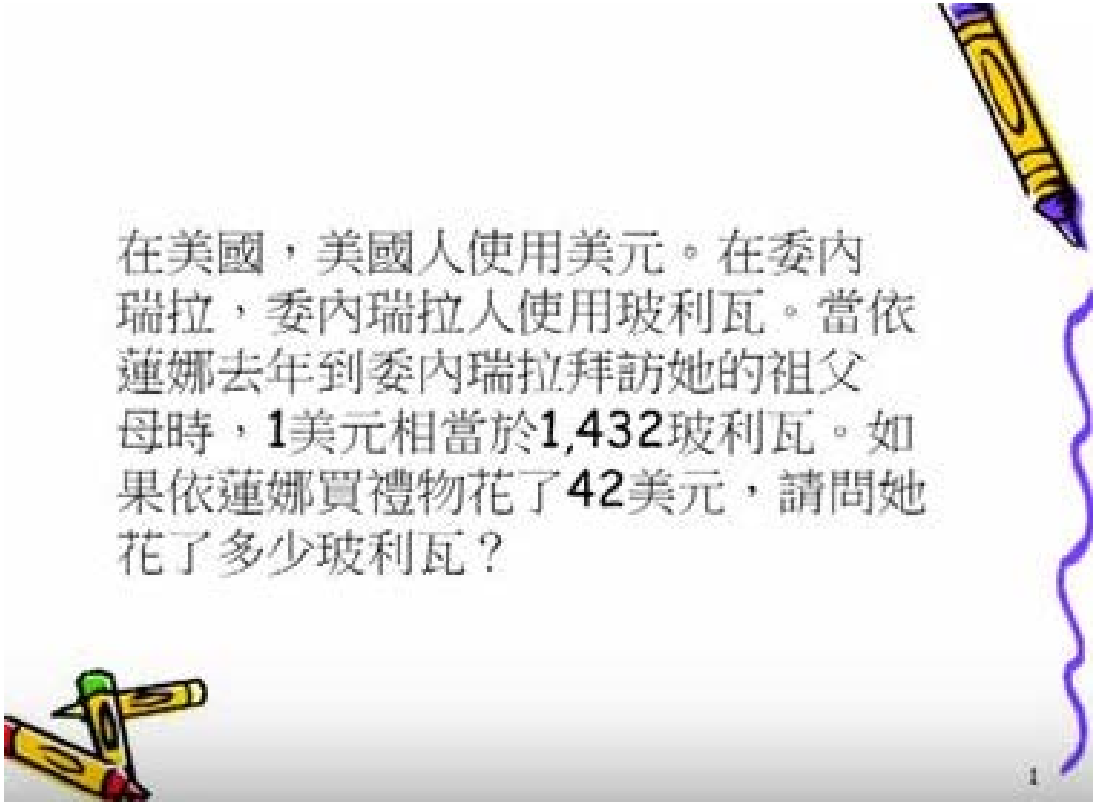
Agenda

- Seeing the Language and Background Knowledge Demands in Mathematics Talk, Text, Tests
- Frontloading Strategies
  - Language
  - Content
- Lesson Framing Strategies
- Interaction/Practice/Language Use Strategies
- Providing Feedback to Learners: What and How to Provide Language Correction
- Resources for Math Teachers of ELLs

Watch and Listen

- Click here to watch

What do you think you are to do?



Look at the Problem. Tell Your Partner What You Think It's Asking

Let's Go Back and Watch, then Discuss:

Did you learn any math?

How did you capture what to do?

(Did visuals help, Did her body language help? Did watching other students help?)

What were you able to do with the print?

How was her rate of speech?

Did you learn any Chinese to help you with other problems you may see on tests?

How Much Did You Capture of the Chinese?

在美國，美國人使用美元。在委內瑞拉，委內瑞拉人使用玻利瓦。當依蓮娜去年到委內瑞拉拜訪她的祖父母時，1美元相當於1,432玻利瓦。如果依蓮娜買禮物花了42美元，請問她花了多少玻利瓦？

In America, Americans use dollars, while in Venezuela, Venezuelans use Bolivar(s). Last year, when Elena went to Venezuela to visit her grandparents, one dollar equaled 1,432 Bolivar(s). Elena spent 42 dollars on buying gift(s). How many (much) Bolivars did she spend?

What affects performance when working in an L2?

--new orthography/writing system to learn to decode

--word knowledge

--rate of speech; redundancy; recycling of terms

--length of the interaction

--scaffolds provided by the teacher

--willingness to keep working despite not understanding most of what is going on

--success with what you are doing

--level of frustration/anxiety

--prior subject matter knowledge

Adapting Teacher Discourse

- Slow down
- Enunciate (without exaggerating)
- Simplify; Be Consistent
- Limit contractions, pronouns, idioms
- Recycle important information (and terms)

Use Gestures

Use Visuals and Graphics

Demonstrate Your Words

Be Dramatic

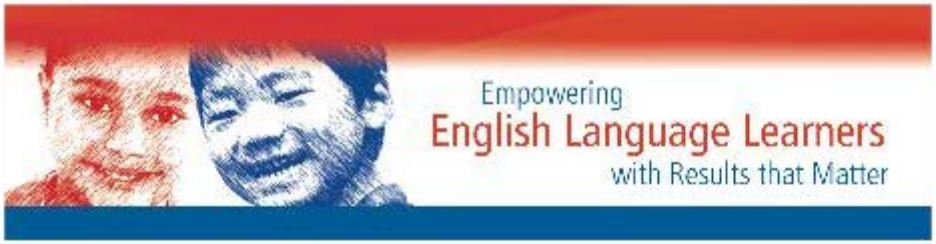
Enhance Your Words

Provide Clear Directions

- **SAY**
- **WRITE**
- **SHOW**

Other EL Student-Friendly Supports

- Be sure the student knows your name
- Establish routines so students know what to expect
- Face the class when speaking
- Avoid slang and explain idioms
- Write legibly; create clear diagrams/visuals
- Repeat important information
- Allow students to audio record lessons
- Provide plenty of wait time
- Post procedures and schedules



5 Principles for Teaching Content to English Language Learners

All children deserve equal access to content—regardless of language level or ability. To achieve this equity, Pearson has developed an instructional framework incorporating five essential principles. This instructional framework is based on the research of numerous language experts, including Dr. Jim Cummins.

1. Identify and Communicate Content and Language Objectives

When presenting content objectives

- **Simplify language** (active voice, use same terms consistently)
- **Paraphrase**
- **Repeat**
- **Avoid** idioms and slang
- **Be aware** of homonyms and multiple-meaning words
- **Clarify** (with similar language, gestures, visuals)
- **Check** for understanding

When working with language objectives focus on

- **Key content vocabulary**
- **Academic vocabulary** (used across the curriculum)
- **Language form and function** (essential for the lesson)

Visit PearsonELL.com

2. Frontload the Lesson

Provide opportunities to frontload or preteach lesson elements

- **Activate prior knowledge** by connecting to students' academic, cultural, or personal experiences
- **Build background** by establishing new vocabulary or unfamiliar facts and concepts
- **Preview text** by reviewing visuals, headings, and/or highlighted text
- **Set a purpose for reading** by clarifying some elementary questions at the end of the lesson
- **Make connections** by helping students see relationships between the lesson and other aspects of their lives

3. Provide Comprehensible Input

Make oral and written content accessible by providing support

- **Visuals** (photos, illustrations, cartoons, multimedia)
- **Graphics** (graphs, charts, tables)
- **Organizers** (graphic organizers, outlines)
- **Summaries** (oral, audio, native language)

(continued on back)

PEARSON

In Your Handouts: 5 Principles of Teaching ELLs

5 Principles of Teaching ELLs

Identify Language Targets and Content Objectives

Frontload the Lesson/Building Background Knowledge

Provide Comprehensible Input

Enabling Language Production/Extending Language

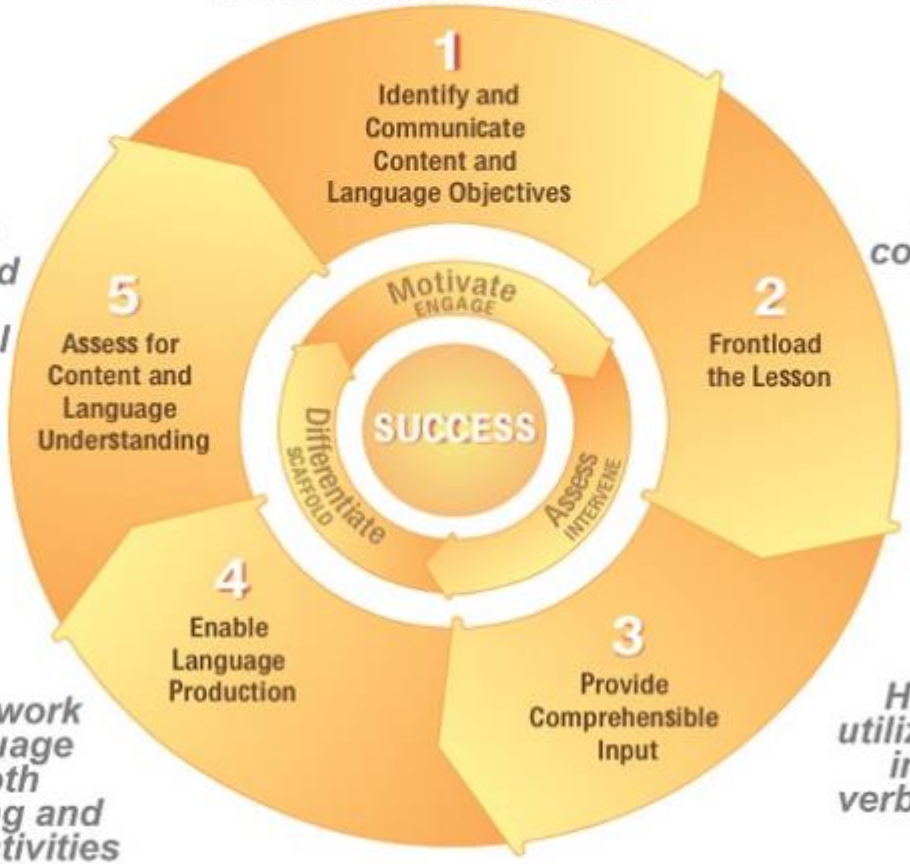
Assessment of Language and Content Objectives

PEARSON LANGUAGE CENTRAL for MATH

Research-based instructional design

5 principles for teaching ELLs

Explicitly stated objectives in each lesson/module



Opening activities connect to and assess prior knowledge

Hands-on activities utilize multiple modes of instruction: visual, verbal, aural, kinesthetic

Pair and group work promotes language practice in both listening/speaking and reading/writing activities

Students express their understanding through speaking and writing activities; reflect on successful strategies

PEARSON LANGUAGE CENTRAL for MATH

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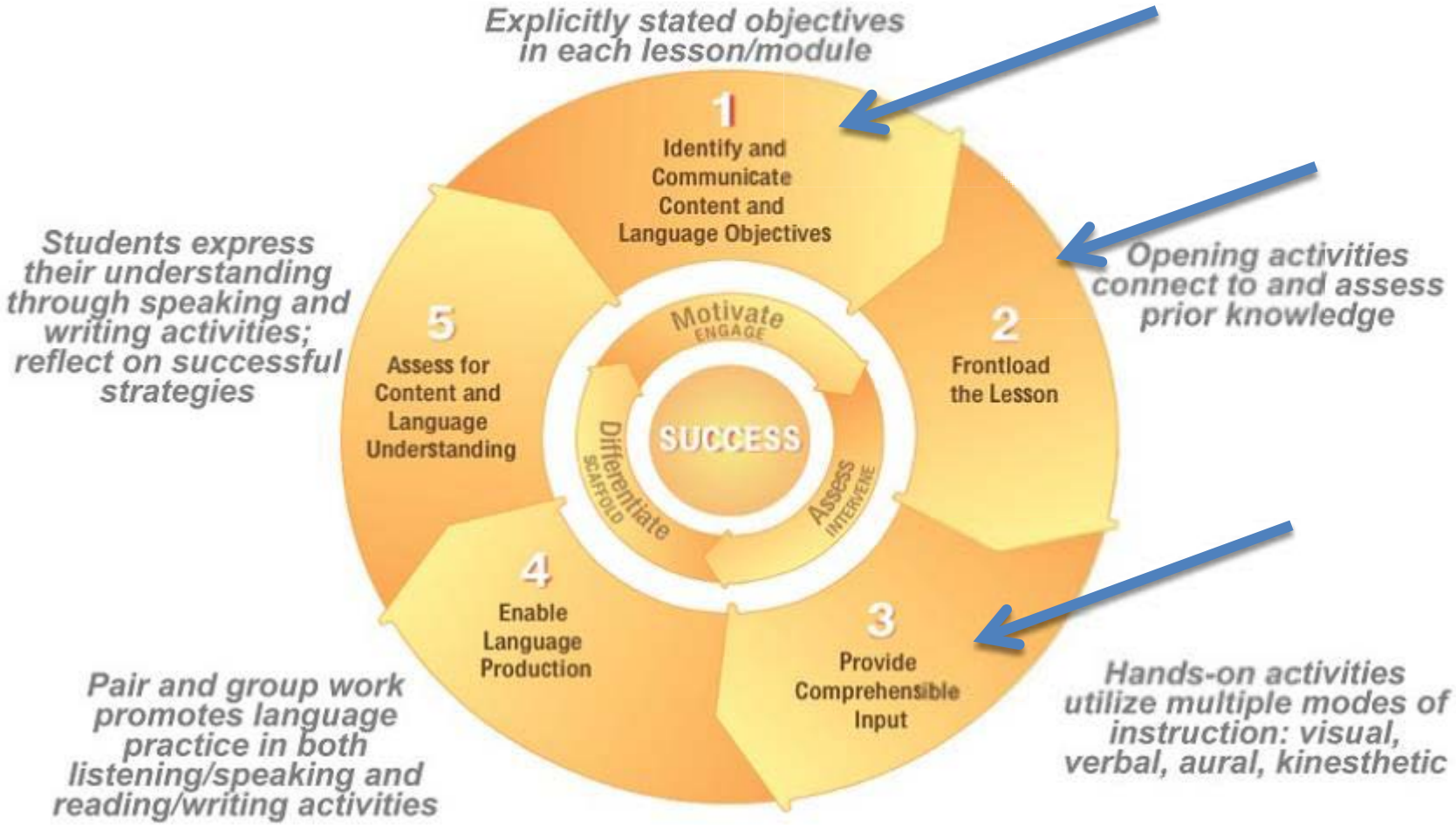

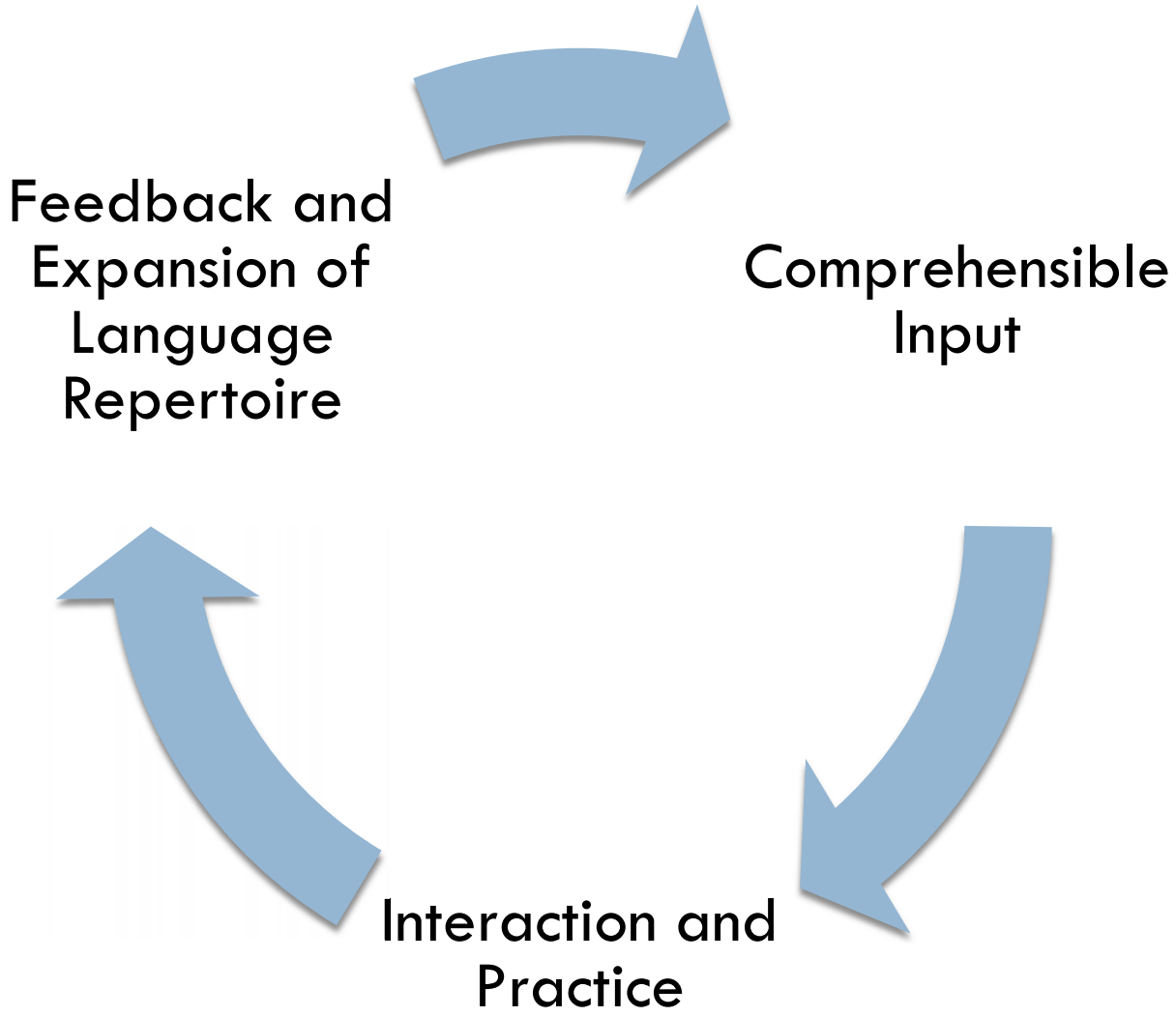

Language Learning Cycle

Planning Language and Content Objectives

Language

- Content-specific Vocabulary
- General Academic Vocabulary
- Structures (grammar)

Content

- Knowledge
- Skills

From NYS Standards; NYC Curriculum

"If only they understood the question, they could answer it. They know the content, they just don't know enough English."

Seeing the Language and Background Knowledge Demands in Math Talk, Text, Tests

Vocabulary Targets:

1. Content-Specific Vocabulary
2. General Academic Vocabulary

Systematically Teach Core Math Vocabulary

Sample Vocabulary Terms

Prepared by Marzano Research Laboratory

May, 2009

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MRL

Absolute error

Absolute value

Acceleration

Add radical expressions

Addition counting procedure

Algebraic function

Angle of depression

Arc

Area under curve

Asymptote of function

Base e

Binary system

Bisect

Bivariate data

Bivariate data transformation

Bivariate distribution

Cartesian coordinates

Categorical data

Central angle

Central limit theorem

Chord

Circle without center

Circular function

Classes of functions

Absolute function

Combination

Completing the square

Complex number

Complex conjugates

Compound event

Compound interest

Conditional probability

Confidence interval

Conic section

Conjugate complex number

Continuity

Continuous probability distribution

Control group

Correlation

Cosine

Critical paths method

Curve fitting

Curve fitting median method

Decibel

Density

Dependent events

Derivation

Dilation of object in a plane

Direct function
Direct measure
Discrete probability
Discrete probability distribution
Divide radical expressions
Domain of function
Empirical verification
Equivalent forms of equations
Equivalent forms of inequalities
Expected value
Experimental design
Experimental probability
Exponent
Exponential function
Factorial
Factorial notation
Fibonacci sequence
Finite graph
Force
Formal mathematical induction
Fraction inversion
Function composition
Function notation
Geometric function
Global/local behavior
Imaginary number

Independent events
Independent trials
Indirect measure
Inductive reasoning
Inflection
Inscribed
Interest
Inverse function
Irrational number
Isometry
Law of large numbers
Law of probability
Limit
Line equation
Line segment
Line segment congruence
Line segment similarity
Line through point not on a line
Linear
Linear system
Log function
Logarithm
Logarithmic funct
Mathematical the
Matrix
Matrix addition

Matrix division
Matrix equation
Matrix inversion
Matrix multiplication
Matrix subtraction
Minimum/maximum of function
Monitor progress of a problem
Monomial
Monte Carlo simulation
Midpoint formula
Measures of dispersion
Natural log
Natural number
Nature of deduction
Negative exponent
Normal curve
Number sub
plot

Point of tangency
Polar coordinates
Polynomial
Polynomial addition
Polynomial division
Polynomial function
Polynomial multiplication
Polynomial solution by bisection
Polynomial solution by sign change
Polynomial solution successive approximation
Polynomial subtraction
Population
Postulate
Powers
Precision of estimation
Probability distribution
Proof paragraph
Protractor
Pythagorean theorem
Quartile deviation
Radical expression
Radical function
Radius
Random sampling technique
Range of function
Rational function

MRL

Use a List

Progress Monitor

Give Credit for Vocabulary Learning

MRL

Tennessee Academic Vocabulary

A Guide for Tennessee Educators

Tennessee Department of Education

Timothy K. Webb, Commissioner
July, 2006
Revised: December, 2007
Revised: July, 2009

Algebra I

Absolute value
Complement of an event
Compound
Conjunction
Direct and inverse variation
Disjunction
Domain & range
Exponential growth (and decay)
Interest (simple and compound)
Irrational numbers
Joint and conditional probability
Law of Large Numbers
Mathematical model
Measure of spread (range, interquartile range)
Midpoint formula
Outlier
Parent function
Pascal's Triangle
Polynomial (binomial, trinomial)
Quadratic formula (including discriminant)
Quantitative and qualitative data
Radicand
Rational expression
Real number properties
Real roots (zeros, solutions, x-intercepts)
Relative frequency
Sequences (arithmetic, geometric, Fibonacci)
Simulations
Subsets of real numbers

Geometry

Altitude
Angle of depression
Angle of elevation
Apothem
Arc
Bisect (bisector)
Central angle
Centroid
Chord
Circumcenter
Circumscribed
Collinear
Concurrent lines
Conditional statement (including converse, inverse, contrapositive,& Biconditional statement)
Construction
Convex & concave polygons
Coplanar
Corollary
Deductive & inductive reasoning
Euclidean & non-Euclidean geometry
Geometric mean
Glide reflection
Incenter
Inscribed
Lateral area
Locus
Negation
Oblique
Orthocenter
Points of concurrency in a triangle
Postulate (axiom)
Proof (formal, two-column, paragraph, flow, coordinate, indirect, counterexample)
Scalar
Secant line
Sector of a circle
Skew lines
Tangent line
Theorem
Trigonometric ratios (sine, cosine, tangent)
Undefined terms of geometry
Vector (magnitude and direction)

Algebra II

Amplitude
Asymptote
Binomial Theorem
Combination
Common ratio (geometric sequence)
Complete the square
Complex conjugate
Complex number
Composition (of functions)
Conic sections (circles, parabola, ellipse, hyperbola)
Empirical Rule
Factorial
Focus (pl. foci)
Independent and dependent events
Inverse of a relation
Logarithm
Normal distribution
Period
Permutation
Piece-wise function
Radian measure
Rational function
Regression equation
Series (arithmetic, geometric, finite, infinite, etc.)
Sigma
Standard deviation
Step function
Synthetic division
Transcendental function
Trigonometric function
Trigonometric identity
Unit circle
Variance

Appendix A – Mathematics Content Words (continued)

Sixth Grade	Seventh Grade	Eighth Grade
algebraic expression	absolute value	adjacent angles
base number	acute triangle	coefficient
circumference	alternate interior/exterior angles	constant
complement	bisector	distance formula: d=rt
convert	combinations	domain
coordinate plane	corresponding angles	formula
diameter	discount	hypotenuse
evaluate	equilateral triangle	lateral area
exponent	experimental probability	legs of a triangle
factorization	exponential notation	linear equation
median	integer	linear inequality
mode	interest	Pythagorean theorem
non-terminating decimal	isosceles triangle	range of a function
numerical expression	negative	rational number
order of operations	obtuse triangle	scatter plot
pi	outcome	scientific notation
plane figure	parallelogram	slope-intercept form
prime factor	permutations	slope
quadrilateral	polygon	solids (prisms, cones, cylinders, pyramids)
radius	positive	standard form (of a linear equation)
reciprocal	proportion	surface area
sequences (arithmetic, geometric, Fibonacci)	quadrant	term
similarity	radical sign	x-y intercepts
simplify	rate	
square units	ratio	
substitution	regular polygon	
supplement	rhombus	
terminating decimal	right triangle	
	scale factor	
	scalene triangle	
	square root	
	theoretical probability	
	transversal	
	trapezoid	
	unit rate	
	vertical angle	

Appendix A – Mathematics Content Words (continued)

Algebra I	Algebra II	Geometry
absolute value function	arithmetic/geometric sequences	altitude
ascending/descending	asymptotes	angle of depression/elevation
binomial	completing the square	angle relationships (complementary, supplementary, etc., expressed algebraically)
degree of a polynomial	complex numbers	arc (measurement, length, major, minor)
difference of squares	composition	central angle
elimination method (for solving a system of equations)	conic sections	chord
factor a polynomial	conjugate (complex)	conditional statements (converse, inverse, contrapositives)
function notation	correlation	congruence
inequalities	curve of best fit	conjecture
intercepts (x & y)	delta	construction (protractor, compass, straightedge)
irrational numbers	discriminant	convex/concave
line of best fit	functions (exponential, polynomial, logarithmic, etc.)	corresponding parts
linear/nonlinear functions (exponential, quadratic, absolute value)	imaginary	counterexample
linear systems	inverse function	deductive reasoning
literal equations	logarithm	distance formula
monomial	matrix	Euclidean/non-Euclidean Geometry
parent graph (linear, absolute value, quadratic, constant)	minimum/maximum (relative, absolute)	inductive reasoning
polynomial	normal distribution curve (Gaussian)	inscribed angles and polygons circumscribed
quadratic equation	parent function (exponential, polynomial, logarithmic)	interior/exterior angles (of a figure)
quadratic formula	radical equation	lateral surface area
rate of change	sigma	median of a triangle
rational expression	standard deviation	midpoint formula
real numbers	synthetic division	polyhedra
relations	three-dimensional coordinate	proof (formal, paragraph, flow, algebraic)
substitution method (for solving a system of equations)	transformation (algebraic)	Pythagorean theorem – area model
trinomial	variance	reflexive, symmetric and transitive properties
	weighted averages	secant line
	zero of a function	tangent line
		theorem/postulate/conjecture
		total surface area
		transformation (reflection, rotation, translation)
		trigonometric ratio (sine, cosine, tangent)

Source: http://sde.state.ok.us/curriculum/BAV.pdf

and Marzano, R. J. (2004). *Building background knowledge for academic achievement: Research on what works in schools*. Alexandria, VA: Association for Supervision and Curriculum Development.

Use the Academic Vocabulary List!(AVL, Gardner)

Choosing **Academic Vocabulary** that appears frequently in academic texts

CLICK $HERE to access the entire list, with hyperlinks to extensive information on each word

1	study	137208	**study** (n) study (v) studied (j) studiously (r) studious (j) *studying* (n) Edu
2	develop	128974	**development** (n) **develop** (v) **developing** (j) *developmental* (j) Edu **developed** (j) developer (n) *developmentally* (r) Edu **underdeveloped** (j) undeveloped (j) *underdevelopment* (n) His redevelopment (n) redevelop (v) *developing* (n) Law
3	group	125012	**group** (n) *grouping* (n) Edu **subgroup** (n) **group** (v) *intergroup* (j) Soc *regroup* (v) His *grouped* (j) Edu *regrouping* (n) Edu
4	system	116141	**system** (n) **systematic** (j) **systematically** (r) *subsystem* (n) Sci **unsystematic** (j)
5	relate	114267	**relationship** (n) **relate** (v) **relation** (n) **related** (j) **relational** (j) **unrelated** (j) **interrelated** (j) **interrelationship** (n) relatedness (n) *interrelation* (n) Hum
6	research	112649	**research** (n) **researcher** (n) **research** (v)
7	social	103635	**social** (j) **socially** (r) *antisocial* (j) Med
8	result	96016	**result** (n) **result** (v) **resulting** (j) **resultant** (j)
9	use	93271	use (v) **use** (n) **user** (n) **useful** (j) used (j) **usefulness** (n) useless (j) **usable** (j) **misuse** (n) *reuse* (v) Sci unused (j) reuse (n) **usefully** (r) *reusable* (j) Sci misuse (v) *usability* (n) Sci **unusable** (j) **useable** (j) *uselessness* (n) Hum misused (j) *uselessly* (r)
10	provide	93212	**provide** (v) *provider* (n) Med **provided** (c) **providing** (c)
11	however	90906	**however** (r)
12	increase	85843	**increase** (v) **increase** (n) **increased** (j) **increasingly** (r) **increasing** (j)
13	experience	79681	**experience** (n) **experience** (v) **experienced** (j) *experiential* (j) Edu inexperienced (j) inexperience (n)
14	level	79201	**level** (n) *level* (j) Edu level (v) **high-level** (j) **leveling** (n) **leveling** (j) leveler (n) leveled (j) *levelly* (r) Soc
15	process	78679	**process** (n) **process** (v) **processing** (n) *processor* (n) Sci *processed* (j) Med *unprocessed* (j) Med *reprocess* (v) Law
16	culture	77470	**culture** (n) **cultural** (j) *culturally* (r) Edu *cross-cultural* (j) Edu **subculture** (n) *intercultural* (j) Edu cultured (j) subcultural (j) uncultured (j)
17	history	77164	**history** (n) **historical** (j) *historian* (n) His **historically** (r) historic (j) prehistory (n) *historicity* (n) Hum+Rel *historicism* (n) Hum
18	active	76010	**activity** (n) **active** (j) activist (n) **actively** (r)

Academic Vocabulary Lists

Corpus of Contemporary American English

Link to an Established Academic Vocabulary List!

Teach words in word families; in semantic networks

http://www.academicvocabulary.info/samples/families.pdf

A Sample Lesson

Lesson 11: Unknown Angle Proofs—Proofs of Known Facts

Classwork

Opening Exercise

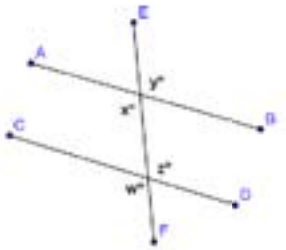
A **proof** of a mathematical statement is a detailed explanation of how that statement follows logically from other statements already accepted as true.

A **theorem** is a mathematical statement with a proof.

Discussion

Once a theorem has been proved, it can be added to our list of known facts and used in proofs of other theorems. For example, in Lesson 9, we proved that **vertical angles are equal in measure**, and we know (from earlier grades and by paper cutting and folding) that **if a transversal intersects two parallel lines, alternate interior angles are equal in measure**. How do these facts help us prove that corresponding angles are equal in measure?

In the diagram to the right, if you are given that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, how can you use your knowledge of how vertical angles and alternate interior angles are equal in measure to prove that $x = z$?



You now have available the following facts:

- Vertical angles are equal in measure.
- Alternate interior angles are equal in measure.
- Corresponding angles are equal in measure.

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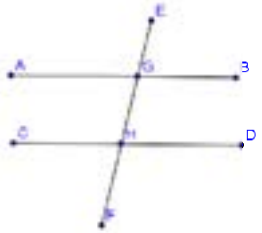
Use any or all of these facts to prove that **interior angles on the same side of the transversal are supplementary**. Add any necessary labels to the diagram below, and then write out a proof including given facts and a statement of what needs to be proved.

Given: $\overline{AB} \parallel \overline{CD}$, transversal $\overline{EF}$

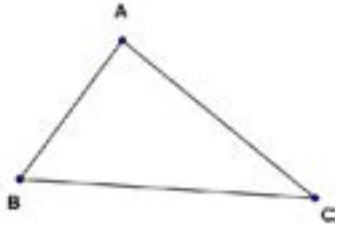
Prove: $m\angle AGH + m\angle CHG = 180°$



Now that you have proven this, you may add this theorem to your list of available facts.

- Interior angles on the same side of the transversal that intersect parallel lines sum to $180°$.

Use any of these four facts to prove that the three angle sum of a triangle is $180°$. (For this proof, you will need to draw an auxiliary line, parallel to one of the triangle's sides and passing through the vertex opposite that side.) Add any necessary labels, and write out your proof.



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Let's review the theorems we have now proven:

- Vertical angles are equal in measure.
- A transversal intersects a pair of lines. The pair of lines is parallel if and only if:
  - Alternate interior angles are equal in measure.
  - Corresponding angles are equal in measure.
- Interior angles on the same side of the transversal add to $180°$. The sum of the degree measures of the angles of a triangle is $180°$.

Side Trip: Take a moment to take a look at one of those really famous Greek guys we hear so much about in geometry, Eratosthenes. Over 2,000 years ago, Eratosthenes used the geometry we have just been working with to find the circumference of Earth. He did not have cell towers, satellites, or any other advanced instruments available to scientists today. The only things Eratosthenes used were his eyes, his feet, and perhaps the ancient Greek equivalent to a protractor.

Watch this video to see how he did it, and try to spot the geometry we have been using throughout this lesson.

https://youtu.be/wrll3aV4eng

Example 3

Construct a proof designed to demonstrate the following:

If two lines are perpendicular to the same line, then they are parallel to each other.

(a) Draw and label a diagram, (b) state the given facts and the conjecture to be proved, and (c) write out a clear statement of your reasoning to justify each step.

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Discussion

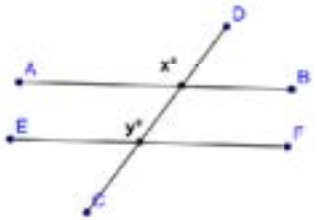
Each of the three parallel line theorems has a converse (or reversing the theorem) that follows.

Original	Converse
If two parallel lines are cut by a transversal, then alternate interior angles are equal in measure.	If two lines are cut by a transversal such that alternate interior angles are equal in measure, then the lines are parallel.
If two parallel lines are cut by a transversal, then corresponding angles are equal in measure.	If two lines are cut by a transversal such that corresponding angles are equal in measure, then the lines are parallel.
If two parallel lines are cut by a transversal, then interior angles on the same side of the transversal add to $180°$.	If two lines are cut by a transversal such that interior angles on the same side of the transversal add to $180°$, then the lines are parallel.

Notice the similarities between the statements in the first column and those in the second. Think about when you would need to use the statements in the second column, that is, the times when you are trying to prove two lines are parallel.

Example 2

In the figure to the right, $x = 1$.
Prove that $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$.



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Problem Set

1. Given $\angle A$ and $\angle Z$ are supplementary and $m\angle A = m\angle Z$.
Prove $\overleftrightarrow{CB} \parallel \overleftrightarrow{DY}$.

2. A theorem that states that **in a plane, if a line is perpendicular to one of two parallel lines and intersects the other, then it is also perpendicular to the other of the two parallel lines.**
Prove this theorem. If (a) Construct and label an appropriate figure, (b) state the given information and the theorem to be proven, and (c) list the necessary steps to demonstrate the proof.

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Choose some vocabulary targets (8-10 terms to really give a workout):

Content Specific Vocabulary

- Proof
- Theorem
- Prove/Proof
- Vertical
- Angle
- Equal measure
- Alternate interior angles
- Corresponding Angles
- Transversal
- Triangle
- (x + y sum to….(sum as verb)
- Vertex
- Draw and Label
- Write out (*write out your proof*)
- Circumference
- Protractor
- Perpendicular

AVL Words; Cross-Disciplinary Academic Terms

- State (Statement) [153]
- Diagram[638]
- Supplement (supplementary)[751]
- Intersect [1325]
- Auxiliary[1457]
- Parallel[565]
- Conjecture[1408]
- Justify [394]
- Converse[912]
- Construct [268]
- Demonstrate [199]

Teach 8-10 vocabulary words well in every "unit"

Cross-disciplinary words from the Academic Vocabulary List

- Choose key AVL words that come up a lot in the unit
- Select 2-3 of these for each unit
- Make sure that you teach these words as part of a word family; teach at least 3 words from the family so kids get experience with different forms of the same word
- Make sure to give practice to these words as much as the content words

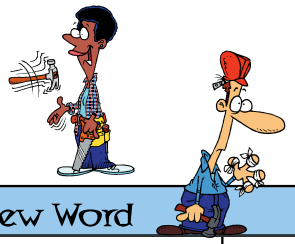
Key Content Words

- Do not teach words students will understand from reading the text (words that are clear from the context; words that are glossed in the text)
- If the word has a cognate students know, skip it
- Make sure to choose different types of words: not just nouns but verbs, adjectives, adverbs
- Not just words but also "lexical bundles"

Forms of a word; Word endings (-al; -ment, -tion, -ing, -ary)

- Mathematical
- Transversal
- Statement
- Explanation
- Information
- Corresponding
- Passing
- Supplementary
- Necessary

Word Builder



Name: _____

Date: _____

Prefix	Word	Suffix	New Word	Meaning
	mathematics	-al	mathematical	
	state	-ment	statement	

Helping Students with Process Words (Communicative Functions)

Responding to Writing Prompts

To write an effective response to a writing prompt, you need to determine more than just your topic, audience, and form. You also need to be able to recognize the key words or directions in the prompt and to know how to respond to those directions appropriately. Familiarizing yourself with the information in the following chart can help you do both of these things.

Direction	*Action*
Describe, Identify, Define	Identify the main equalities or distinguishing characteristics of your subject, using specific facts or sensory details.
Recount, Narrate, Relate, Tell About	Present the sequence of events in a story or the steps in a process in chronological order. Use vivid verbs and sensory details.
Discuss	Identify the key points or relationships, backing these up with examples, quotations, comparisons, and other details.
Explain	Present the main points of or important steps in whatever you have been asked to explain, using facts, examples, and reasons to clarify what you mean.
Compare, Contrast	Show the ways in which two or more things are alike and the ways in which they differ, using examples and other details to support similarity and difference you point out.
Analyze, Evaluate, Review	Examine the main qualities of your subject to arrive at a conclusion about some aspect of it, such as how well the parts function together as a whole.
Show Causes and Effects	Present the reasons for and the results of a particular event or situation, using specific details to clarify precisely what happened.
Persuade, Convince, Express Your Opinion	State your point of view and support it with facts, statistics, examples, quotations, and other sound evidence.
Interpret	Explain in your own words the meaning of whatever you've been asked to interpret, supporting your ideas with facts and other details.
Summarize	Present a condensed version of a story or a process by relating only the main events or steps and showing how one leads to the next. Do *not* include supporting details or other types of elaboration.

THE TWELVE WORDS[2]

Educators have identified 12 words that sometimes trip up students when they are responding to questions in writing or speaking and when taking a test[3] Understanding and using these 12 words are critical for student success from upper elementary to post-secondary studies. Schools are encouraged to develop school-wide efforts to assure that students become familiar with these key words.

DESCRIBE	Describe means to: • tell about something • show • illustrate • list the attributes of a thing	When you *describe* a thing, you're thinking about "what does it look like?" When you are *describing* something in your writing or speaking, you are telling about that thing in a way that paints a clear picture of the topic.
EXPLAIN	When you explain you: • restate something with more details • retell in order • give important information • tell more about something	When you *explain* something think about a teacher who is teaching you how to do something. They want to describe the process so that you know what to do first, next, and last. When you are asked to *explain* something you are being asked to share something with enough clarity and detail so that the recipient easily understands it.
TRACE	Trace means to: • outline • sketch or draw • map out • copy • follow from the beginning	When you see the word *trace* think of a pencil tracing along a line. It follows the line from the beginning to the end. So when you trace something you are thinking, and then writing or speaking about the subject from the beginning to the end.
SUPPORT	Support means to: • give facts for something • explain why • prove it • use examples from something • give someone reasons • back up your point of view	When you are asked to *support* something, think about a person taking a position about something and then giving more information to back up his/her position. In writing or speaking when you *support* something you include a number of facts and/or reasons to support of your position.
PREDICT	To predict is to: • foretell • forecast • tell what you think will happen • describe what you see might be coming	When you *predict* something, you are stating what you expect to happen in the future, A person giving a weather forecast is *predicting*. When you see the word *predict* ask: What might happen next? When you are asked to *predict* in writing or speaking, you will be telling what you think will be happening in the future.
COMPARE	To compare means to: • think about what things have in common • check for likenesses, similarities • match up things	When you *compare* things, ask "how are they alike?" or What do they have in common? When you are asked to *compare* things in your writing or speaking, you need to think about how the things are the same, or how they are alike.

[2] We believe that the source of the Twelve Words is Larry Bell, educational consultant.

[3] To view a simple slide show on the twelve words above, go to: http://www.owensboro.k12.ky.us/edtech/12words/12words.htm

The Twelve Words (continued)

CONTRAST	To contrast means to: • identify how things are unalike, or opposite • check for differences	When you *contrast* things ask yourself "How are they different?" Apples and oranges are fruits but there are many differences. When you are *contrasting* things in your writing or speaking you are describing how they are different from each other.
SUMMARIZE	Summarize means to: • recall • shorten • a brief description • list the main points	Do you see the word "sum" in *summarize*? Think about adding or summing up a thing. When you are asked to *summarize*, you create a short piece about something by putting it in your own words.
INFER	Infer means to: • suggest • conclude • fill in the blanks • decide the meaning of	When you hear the word *infer* ask yourself, "What does this make me think? What is the author trying to tell me without using the exact words?" Infer can mean to read between the lines. In writing or speaking when you are asked to *infer*, it means that you think about what can you suggest or conclude from what you have read?
ANALYZE	Analyze means to: • take it apart • break it down • examine • explore • investigate	When you *analyze* something you look at it closely. When you think of *analyze* think of someone taking a bicycle apart to explore its parts and take a closer look at it. When you *analyze* something you are breaking it down to take a closer look at it before you speak or write about it.
EVALUATE	Evaluate means to: • grade • rank • rate • review a performance • decide what is good or bad about something • judge the quality	Think about the athletes at the Olympics. The judges *evaluate* what is good or bad about their performance and give them a ranking or grade. When you are asked to *evaluate* something you think about what is good or bad about that particular thing and then write or speak about it.
FORMULATE	Formulate means to: • build • add up • plan • construct • make it	When you f*ormulate* something, you put it all together in a plan with details. When you hear the word *formulate*, think about putting a puzzle together. When you are asked to *formulate* in your writing or speaking you will create a piece that constructs or pulls things together into a plan.

Prepositions:

e.g. In the diagram to the right….

Find four solutions to the equation $(X^2 - 9)(x^3 - 8) = 0$

How vertical angles and alternate interior angles are equal in measure to prove….

What is the degree of your polynomial?

Can you find a rule that relates the multiplicities of the zeros to the degree of the polynomial function?

Preposition List

Place

above
across
against
among
around
at
behind
below
beneath
beside
beyond
by
in
inside
into
near
next to
off
on
out
outside
over
through
throughout
toward
under
up
upon
with
without

Time

after
around
at
before
between
by
during
for
from
in
on
since
to
until
within

Relationship

besides
except
like
of

Lexical bundles/Phrases/ Transitions:

FIGURE 4.1 Common Transition Words

Many transition words or phrases can be used as Sentences Starters, while some connect ideas inside a sentence. They can be provided to English learners on posters to support oral language and in sentence frames to support written communication.

Purpose	Transition Words			
Giving a definition	is equal to	means	refers to	is synonymous with
	is the same as	in other words	consists of	in fact
Providing an example	for example	for instance	such as	is like
	including	to illustrate		
Suggesting more ideas	furthermore	also finally	another	moreover
Sequencing	first…second	next	initially	before
	preceding	when	finally	after
	following	as	not long after	now
Comparing	same as	just like/as	in the same way	in comparison
	not only…but also	as well as	similarly	
Contrasting	different from	as opposed to	instead of	in contrast
	however	but	although	yet
	while	on the other hand		
Showing cause and effect relationships	because	as a result of	may be due to	since
	consequently	this led to	so that	nevertheless
	in order to	effects of	for this reason	if … then
	therefore	thus		
Describing problems and solutions	one answer is	one reason is	a solution is	the problem is
	the question is			
Expressing an opinion or conclusion	I think	I believe that	I predict that	I suggest that
	I conclude that	I deduce that	I speculate that	in my opinion
	I agree with ___ that			
Reporting findings or outcomes	I/We found that	I/We learned that	I/We discovered that	I/We observed that

Source: Adapted with permission from English Learners and the Language Arts (ELLA). (2003). San Francisco: WestEd.

Components of Academic Language

Bricks: **vocabulary**

Mortar:
Grammar/syntax/**form**

Foundation:

Language functions

Zwiers, 2008

Paul Quaglio, University of NY at Cortland
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The Demands of Mathematics

Write two functions using the expressions on each side of the equation $6+1 = (x+7)^2$. **Graph** the functions.

Add whole numbers and add fractions. **Compare** with the estimate. **You** can also find the sum using a model.

In the shopping mall giveaway, each store represents an outcome. If you want to pick a food store, the 25 food stores are favorable outcomes. If each outcome has an equal chance of happening, you can find theoretical probability using this formula.

Lexical bundles: you can use a formula **to find the area of** a square or the volume of a cube.

Table 1. Features that differentiate *Math* from Soc. Studies & Science

Feature	Math	Soc St	Science
2nd per pronoun	19300	1700	14100
imperatives	**15421**	**1262**	**3767**
private verb	15200	9100	8900
***Wh*-questions**	5800	400	1400
public verbs	5400	2900	1200
prediction modals	4100	3000	2900
time adverbials	4100	2800	3200
Subordination (condition)	3400	600	2500
***Wh*-clauses**	2200	500	800
be as main verb	1600	700	1200
***that*-deletion**	1400	1100	700
stranded prep	1400	500	600
Features associated with involved registers			
present tense	91000	32000	93300
possibility modals	6218	3391	9151

What features do you see in the text below?

Consider the polynomial function $Qx=x^6-3x^5+4x^3-12x^2+x-3$.

a) Divide Q by the divisor $(x-3)$ and rewrite in the form $Q(x)=$(divisor)(quotient) + remainder.

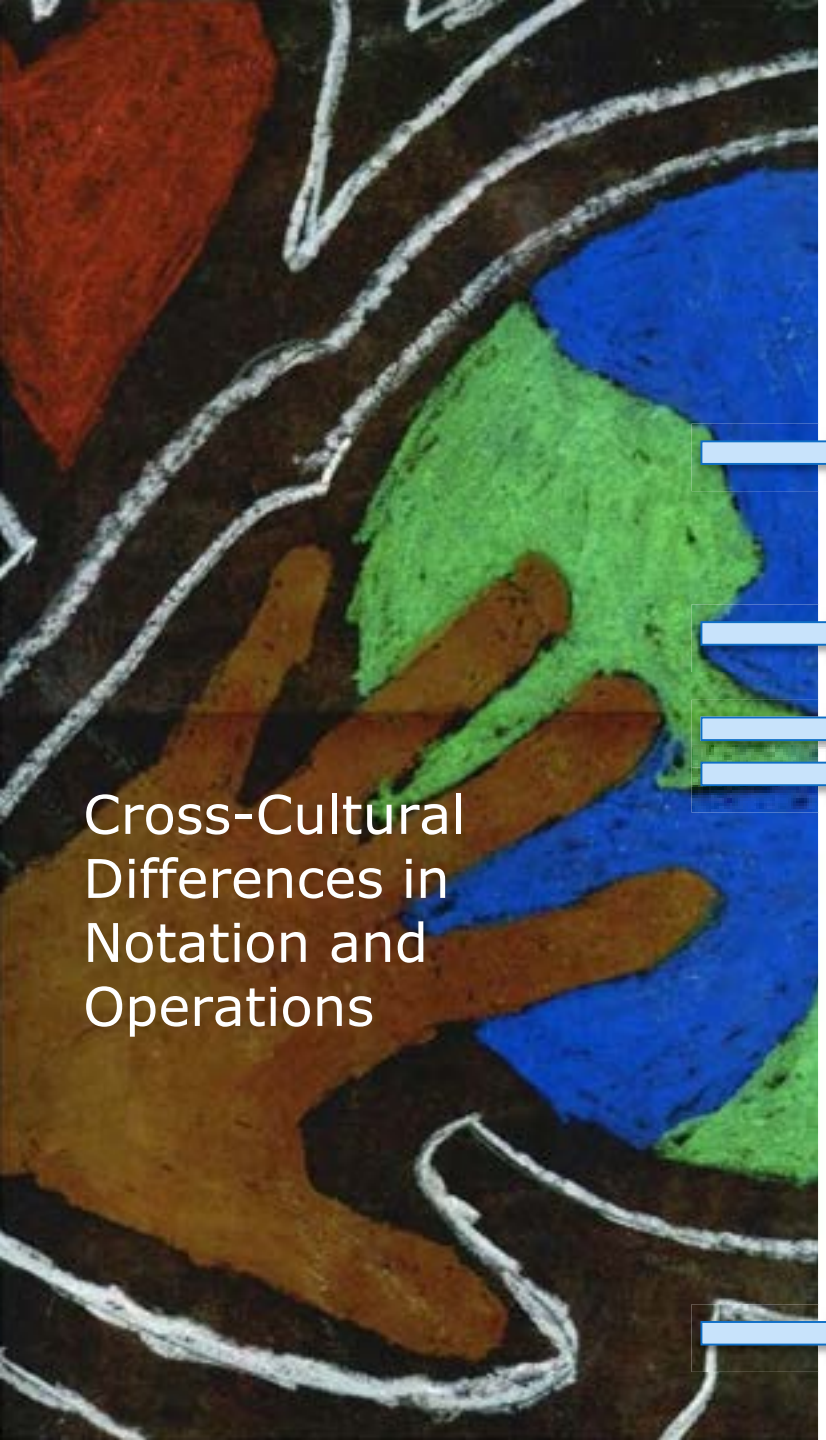
b) Evaluate $Q(3)$.

In the diagram to the right, if you are given that $\overline{AB} \parallel \overline{CD}$, how can you use your knowledge of how vertical angles and alternate interior angles are equal in measure to prove that $x=w$?

- Imperatives (commands)
- Private verb (think, consider)
- WH questions
- Prepositions
- Prediction modals
- Time adverbials
- Subordination (conditions)
- WH or that clauses
- Second person pronoun (you)

Mathematics Discourse Features

- conceptually packed.
- high-density of important words.
- require up-and-down as well as left-to-right eye movements.
- require reading-rate adjustment.
- require multiple readings.
- use numerous symbolic devices.
- contain a great deal of technical language with precise meaning.



Cross-Cultural Differences in Notation and Operations

FIGURE 4.2. Mathematical Notational and Procedural Comparisons Between U.S. and Latin American Countries

	United States	Latin American Countries
Reading Numbers	Numbers with 10, 11, or 12 digits are designated as billions. 8,000,000,000 is read "eight billion" 8,000,000,000,000 is read "eight trillion"	Numbers with at least 13 digits are designated as billions. 8,000,000,000 is read "eight thousand million" 8,000,000,000,000 is read "eight billion"
Separation of digits in large numbers	Comma	Decimal point, space, comma, apostrophe, or semicolon
Negative Numbers	Preceding negative sign (–)	Preceding negative sign (–) or bar over the number
Repeating Decimals	Bar over repeating digits or ellipsis following digits $0.32 = 0.3\overline{232}...$	Arc over repeating digits $0.\overset{\frown}{32}$
Decimal Fractions	Decimal point 4.56	Comma 4,56
Operation Symbols	Colon denotes division primarily in ratios.	Colon is one of four symbols denoting division.
Angle Notation	Angle symbol to the left of angle name	Angle symbol above angle name
Measurement	US Customary	Metric
Prime Factorization	Factor Tree	Vertical Line
Division of Fractions	Invert second fraction and then multiply	Cross-multiply
Least Common Multiple	Use prime factorization	Multiply common prime factors and the prime factors that appear in each number
Subtraction Algorithm	Renaming method $42 - 19 = (30 + 12) - (10 + 9)$	"Equal additions" method $42 - 19 = (40 + 2) - (10 + 9)$ $= (40 + 12) - (20 + 9)$
Division Algorithm	"Long" division	Rely more on mental mathematics or "short" division
Parentheses	Evaluate within parentheses first	Use distributive property
Algebraic Equations	Perform operations on both expressions of the equation $x + 35 = 75$ $x + 35 - 35 = 75 - 35$ $x = 40$	Mentally find missing number in each operation $x + 35 = 75$ What plus 35 equals 75? $40 + 35 = 75$

For Your Reference

Academic Language of Mathematics

TEXT/TALK FEATURES

- conceptually packed
- high density of unique words with specific meanings
- great deal of technical language with precise meanings
- requires multiple readings
- requires a reading rate adjustment because text must be read more slowly than natural language texts
- uses numerous symbols
- many charts and graphs

MAJOR TEXT STRUCTURES AND FEATURES OF TALK

cause and effect; comparisons; logical or chronological sequence

SUBJECT MATTER–SPECIFIC VOCABULARY

e.g., *divisor, denominator, integer, quotient, coefficient, equation, protractor, place value, proper/improper fraction*

WORDS USED IN NEW WAYS

e.g., *table, column, variable, carry, irrational/rational, mean, factor, term, expression, odd, set*

MULTIPLE WAYS OF SAYING THE SAME THING (SYNONYMS)

e.g., *add, plus, combine, and, sum, increased by, total; subtract from, decreased by, less, minus, differ, less than, have left*

COGNATES (SPANISH/ENGLISH)

e.g., *base/base; centímetro/centimeter; columna/column; concepto/concept; número/number; ordinal/ordinal; grupo/group; identificar/identify; secuencia/sequence; ángulo/angle; círculo/circle; diferencia/difference; dividir/divide; línea/line; multiplicar/multiply*

PHRASES WITH SPECIFIC MEANINGS; LEXICAL BUNDLES

e.g., *least common multiple, standard deviation, square root, a quarter of, divided by vs. divided into, as much as, common factor, the size of the, greater than or equal to, not more than*

TRANSITION WORDS; LOGICAL CONNECTORS

if . . . then, if and only if, because, that is, for example, such that, but, consequently, either

COMMON COMMUNICATIVE FUNCTIONS

following directions in a sequence, show, tell, ask and answer factual questions, predict, explain, justify, hypothesize, conjecture

HELPFUL READING/WRITING SKILLS AND STRATEGIES

adjust reading rate, reread difficult text, confirmation checks/summarize as you go, take notes while reading, use graphs, number lines, and charts to complement the understanding of text

Planning Your Language Objectives

Language Form

- Sounds of Language
- Form of words (nouns, pronouns, verbs, adjectives, adverbs)
- Structure of sentences; sentence patterns
- Length, Complexity of Sentences
- Connectors; Cohesive ties

Language Content & Use

- Semantics (vocabulary; phrasing; tone)
- Pragmatics (intended meanings; language as used in different contexts)

Language Functions

- Kinds of communicative tasks students must accomplish (name, tell, compare, describe, explain, apologize, insult, request)

Language Style

- Formal vs. informal/casual
- Social vs. academic
- Colloquial Language (idioms; common similes, metaphors)

Suggested Language Objectives:

Language Forms

- Sounds of Language (*th* in theorem, *v* in vertex)
- Parts of Speech (**prepositions, commands, comparative/ superlative adjectives**)
- Sentence Patterns (**clauses, modals (could, may), possibility—if…then, given….**)
- Length, Complexity of Sentences

Language Functions

- Kinds of communicative tasks students must accomplish:
  - Draw and label,
  - find,
  - consider,
  - compare,
  - determine,
  - explain,
  - state, etc.

Common communication in mathematics classrooms

Functions

- discussing ideas and asking questions,
- summarizing instructional key ideas or defining a term
- following and giving instructions,
- recounting how a problem was solved,
- explaining thinking/reasoning aloud,
- giving reasons for a response,
- showing problem solving steps to display knowledge on tests
- describing procedures to follow

Make Sentence Frames for These Functions

Frontloading Strategies:
Teaching Background Knowledge and Key Vocabulary

Background Knowledge/Life Experience Demands: Algebra 1 Regents

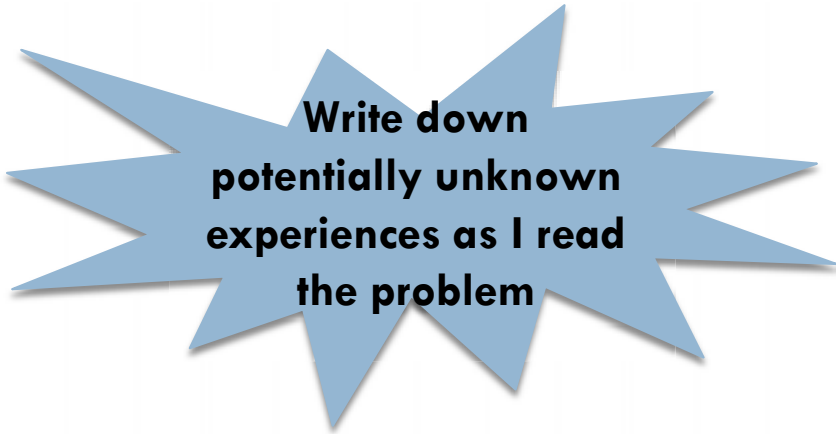
1) Joy wants to buy strawberries and raspberries to bring to a party. Strawberries cost $1.60 per pound and raspberries cost $1.75 per pound. If she only has $10 to spend on berries, which inequality represents the situation where she buys x pounds of strawberries and y pounds of raspberries?

(1) $1.60x + 1.75y \leq 10$

(3) $1.75x + 1.60y \leq 10$

(2) $1.60x + 1.75y \geq 10$

(4) $1.75x + 1.60y \geq 10$



2) On the main floor of the Kodak Hall at the Eastman Theater, the number of seats per row increases at a constant rate. Steven counts 31 seats in row 3 and 37 seats in row 6. How many seats are there in row 20?

(1) 65 (3) 69 (2) 67 (4) 71

3) Which situation is not a linear function?
(1) A gym charges a membership fee of $10.00 down and $10.00 per month.
(2) A cab company charges $2.50 initially and $3.00 per mile.
(3) A restaurant employee earns $12.50 per hour.
(4) A $12,000 car depreciates 15% per year.

4) The amount Mike gets paid weekly can be represented by the expression 2.50a + 290, where a is the number of cell phone accessories he sells that week. What is the constant term in this expression and what does it represent?
(1) 2.50a, the amount he is guaranteed to be paid each week
(2) 2.50a, the amount he earns when he sells a accessories
(3) 290, the amount he is guaranteed to be paid each week
(4) 290, the amount he earns when he sells a accessories

5) There are two parking garages in Beacon Falls. Garage A charges $7.00 to park for the first 2 hours, and each additional hour costs $3.00. Garage B charges $3.25 per hour to park.

When a person parks for at least 2 hours, write equations to model the cost of parking for a total of x hours in Garage A and Garage B.

Things ELLs may not have experience with:

- American names (Joy, Steven, Mike, Dylan)
- Foods common in US (strawberries, raspberries)
- Activities common in US (parties, gym memberships, cabs, parking garages; buying&selling cell phone accessories, campgrounds)
- Measurement/Monetary systems in US (pounds, miles, currency and coins)
- Places (Kodak Hall at the Eastman Theater)
- How public spaces are organized (seats in rows)
- Ways we value merchandise (automobile depreciation).

How to Help ELLs over cultural and linguistic hurdles:

- Make sure they know about pounds (lbs.), dimes, quarters, feet, cubic feet, miles
- Help recognize names of people (capitalized in sentences doesn't matter if male or female (but they may get a clue in the sentence if it says "he"/"she")
- Teach symbols like $, % ≥ ≤ and the words for these (e.g. dollars, percent…) as items may contain symbols or words
- Make sure they capture/look for negation (is NOT)
- Teach lexical bundles like "at least", "for a total of", "If….only", "must be true", "to the nearest tenth", "approximate value"

More things you can help with:

- Capturing the meaning of prepositions common in math: inside, of, in, into, for, on, at, per
- Practice deciphering the meaning of "if" clauses
- Help with infinitives: to be paid (e.g. (wants to be paid, to bring, to spend
- Help with the command words in problems: Justify, state, explain, determine, display, represent, show, model, write…
- Make sure they capture comparatives and superlatives (greater, greatest)
- Help them capture verbs of possibility (modals): must be, can be

Offer Templates Students Can Use

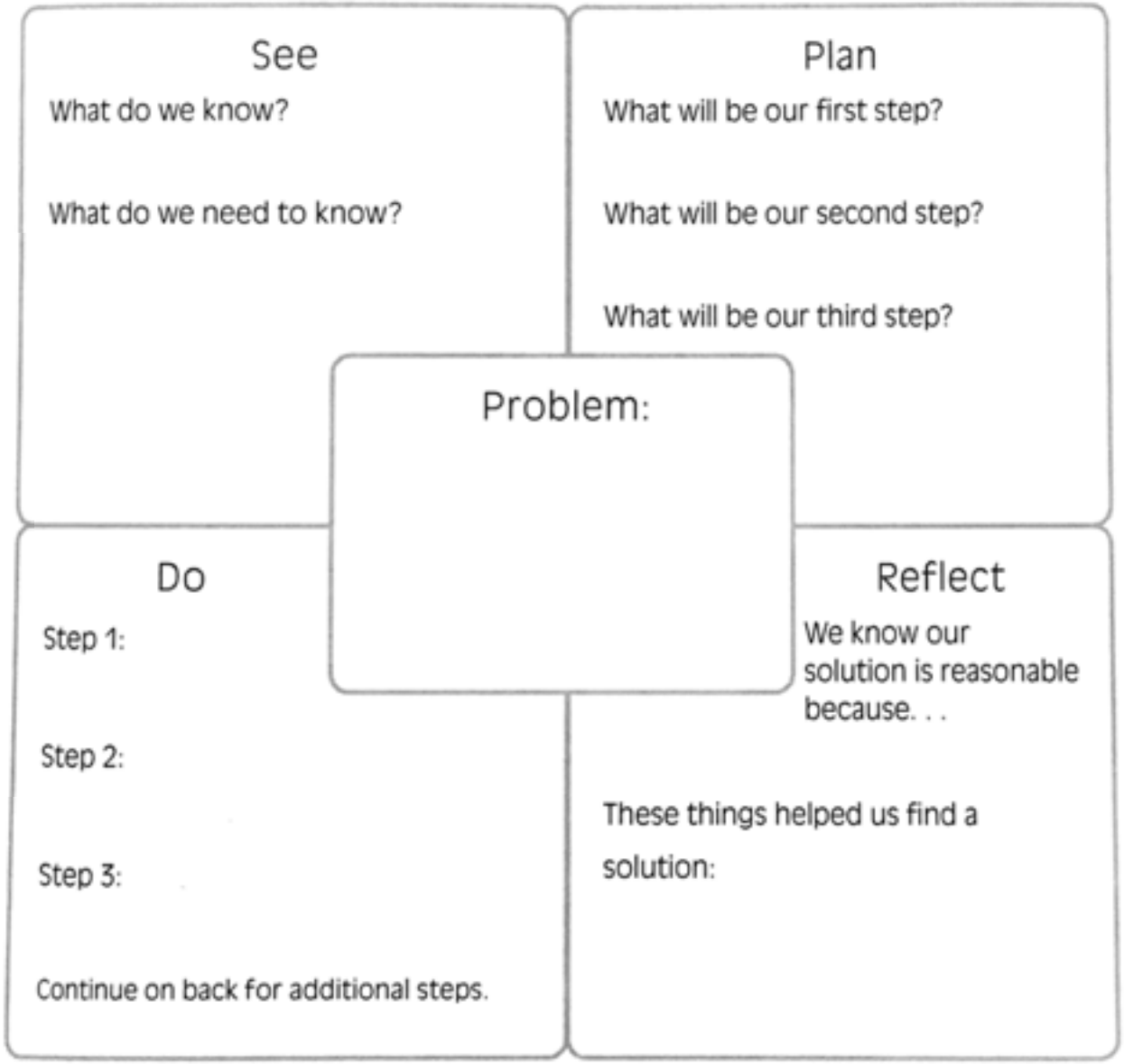


Figure 4.14: Sample See-Plan-Do-Reflect model.

Connect to and Assess Prior Knowledge—Ask:

- What do you see in the lesson that might be unfamiliar to your ELLs?
- What prior knowledge is needed to understand this lesson? (concepts upon which the lesson depends?)

Previous Mathematics Learning Survey:

Name:
Your Home Country:
Your Native Language:

Did you study mathematics in your home country? ☐Yes ☐No	When did you begin learning mathematics in your home country? (at what age/grade?)
Have you studied mathematics in the US before this year? ☐Yes ☐No **How many years have you studied mathematics in the US?**	**Where did you study mathematics in the US?** **State(s):** **School Districts:**

What subjects have you learned in your home country or in the US?
☐Arithmetic (General Mathematics)
☐Algebra
☐Geometry
☐Statistics and Probability
☐Calculus
☐Other: Please specify _____

I learn mathematics by: (check all that apply)
☐reading the textbook
☐listening to the teacher
☐demonstrations by the teacher
☐doing exercises in class
☐practicing at home/completing homework
☐working together with my classmates
☐asking friends/siblings for help outside of school
☐translating the work to my home language
☐learning mathematics vocabulary
☐memorization
☐using drawings, illustrations, visual displays
☐staying after school for extra help/after school tutoring
☐other; please specify: _____

Do you enjoy mathematics?
☐Yes ☐No

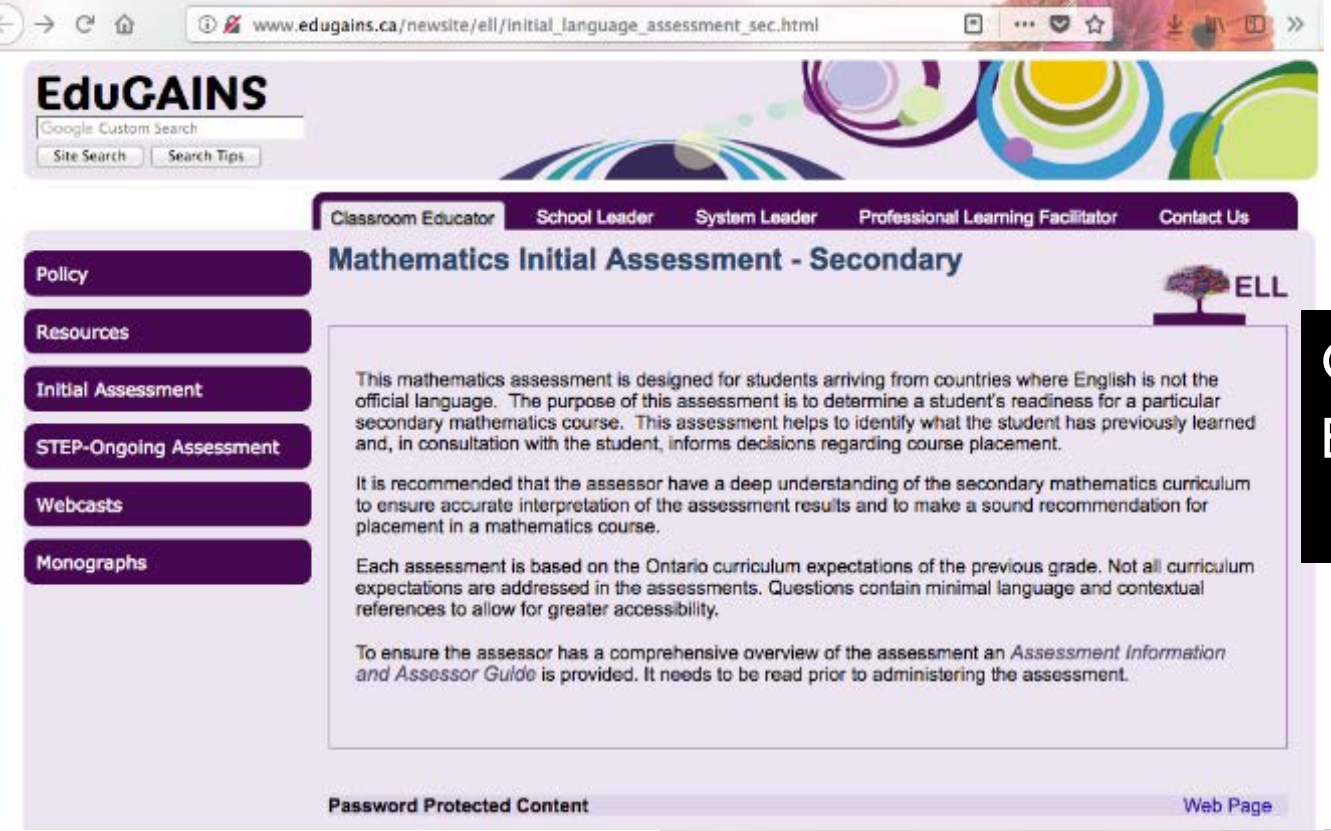
What do you like most about mathematics:

What is most difficult for you in learning mathematics?

Survey Student Prior Learning

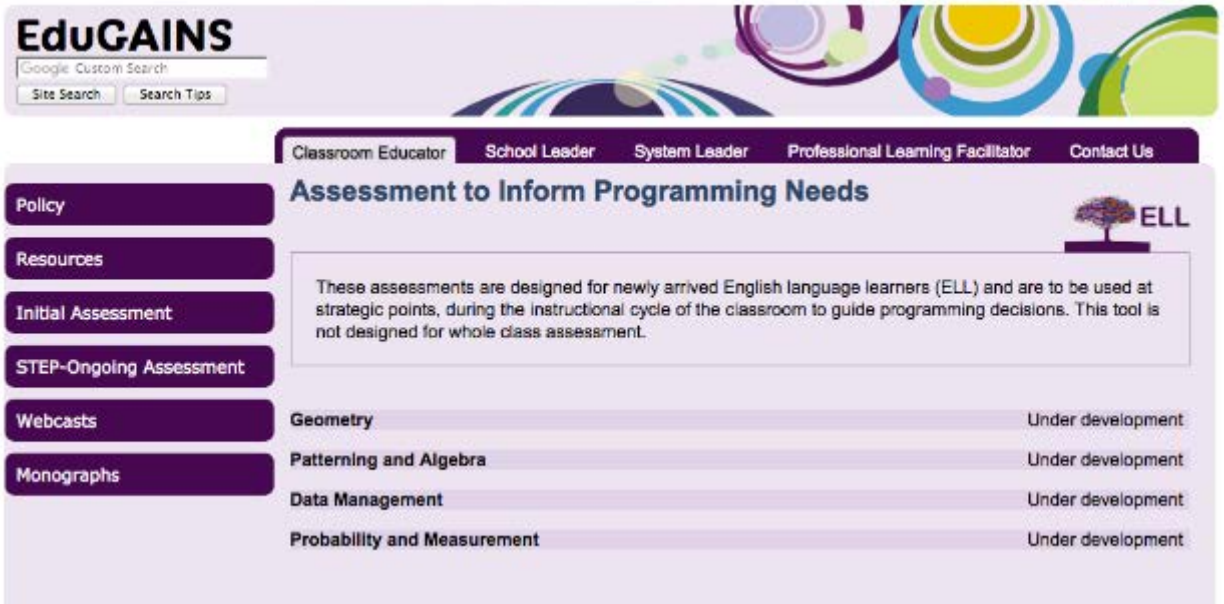
Do quick assessments (quick writes, exit slips in L1 or L2)

Review student records to see what math courses they have had



Ontario Ministry of Education

http://www.edugains.ca/newsite/ell/index.html



In order to choose 8-10 terms to really give a workout (see next slide):

Content Specific Vocabulary

- Proof
- Theorem
- Prove/Proof
- Vertical
- Angle
- Equal measure
- Alternate interior angles
- Corresponding Angles
- Transversal
- Triangle
- (x + y sum to….(sum as verb)
- Vertex
- Draw and Label
- Write out (*write out your proof*)
- Circumference
- Protractor
- Perpendicular

AVL Words; Cross-Disciplinary Academic Terms

- State (Statement) [153]
- Diagram[638]
- Supplement (supplementary)[751]
- Intersect [1325]
- Auxiliary[1457]
- Parallel[565]
- Conjecture[1408]
- Justify [394]
- Converse[912]
- Construct [268]
- Demonstrate [199]

Find Out What Key Terms Students Know:

Place "candidate" words in first row

Have students complete before you start the unit

VOCABULARY IN CONTEXT

word	I recognize it in context, I think it has something to do with	I have never seen the word before, so to learn about it I will	I have heard of the word, but I don't know what it means. To understand it, I am going to	I know the word, it means
				Use L1 or L2

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Relevant Vocabulary Terms

In the definitions below, the symbol *R* stands for the set of real numbers.

FUNCTION: A *function* is a correspondence between two sets, *X* and *Y*, in which each element of *X* is assigned to one and only one element of *Y*.

The set *X* in the definition above is called the *do* ...*or* *image*) of the function is the subset ..., de... property: *y* is an element of *f(X)* if ...nd

If *fx=x2* where *x* can be any real num... ...d *R*), and the range is the set of nonne...

POLYNOMIAL FUNCTION: Given a po... *function in one variable* is a functio... domain, *f(x)* is the value found by ... variable symbol in the polynomia...

It can be shown that if a function ... negative integer *n* and collection ... the function satisfies the equation

fx=anxn+an-1xn-1+…+a1x+a0,

for every real number *x* in the domain, ... *polynomial function*. The function *fx=3x3+4x2+4x+7*, where *x* can be any ... written in standard form.

DEGREE OF A POLYNOMIAL FUNCTION: The *degree of a polynomial function* is the degree of the polynomial expression used to define the polynomial function. The degree of $fx=8x3+4x2+7x+6$ is 3, but the degree of $gx=x+12-x-12$ is *1* because when *g* is put into standard form, it is $gx=4x$.

CONSTANT FUNCTION: A *constant function* is a polynomial function of degree 0. A constant function is of the form $fx=c$, for a constant *c*.

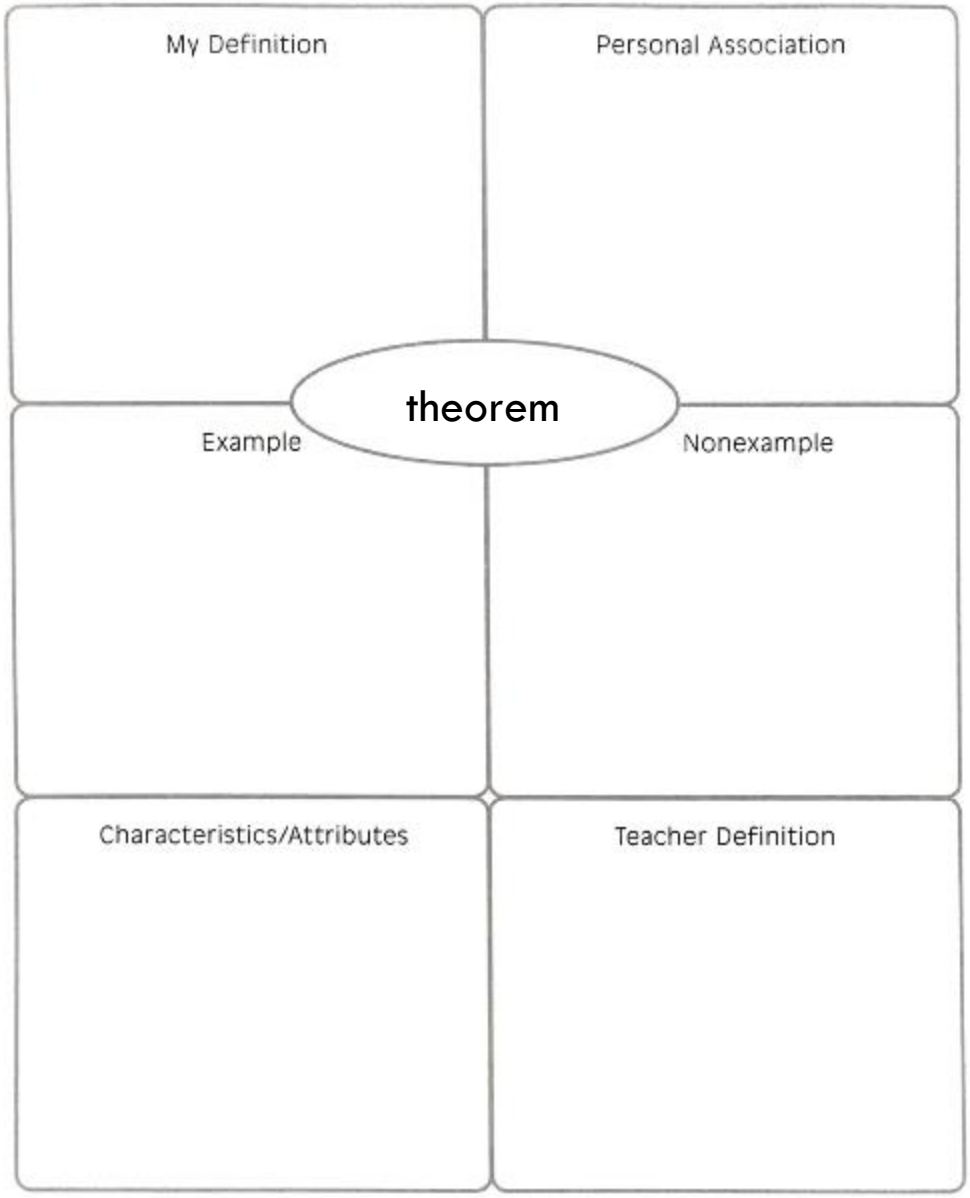
LINEAR FUNCTION: A *linear function* is a polynomial function of degree *1*. A linear function is of the form $fx=ax+b$, for constants *a* and *b* with $a\neq0$.

QUADRATIC FUNCTION: A *quadratic function* is a polynomial function of degree *2*. A quadratic function is in *standard form* if it is written in the form $fx=ax2+bx+c$, for constants *a*, *b*, *c* with $a\neq0$ and any real number *x*.

CUBIC FUNCTION: A *cubic function* is a polynomial function of degree 3. A cubic function is of the form $fx=ax3+bx2+cx+d$, for constants *a*, *b*, *c*,*d* with $a\neq0$.

ZEROS OR ROOTS OF A FUNCTION: A *zero* (or *root*) of a function $f: R\rightarrow R$ is a number *x* of the domain such that $f(x)=0$. A zero of a function is an element in the solution set of the equation $f(x)=0$.

Vocabulary Organizer



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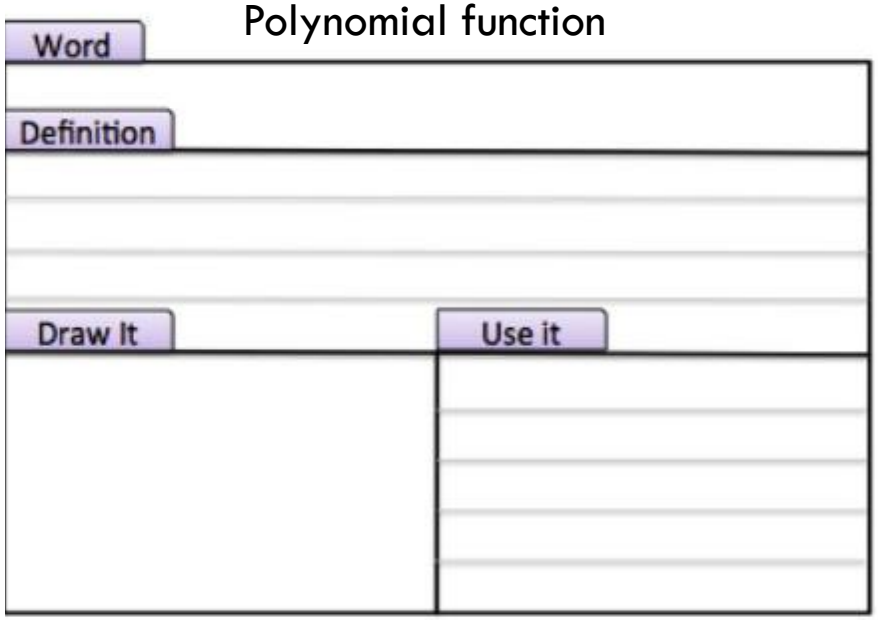
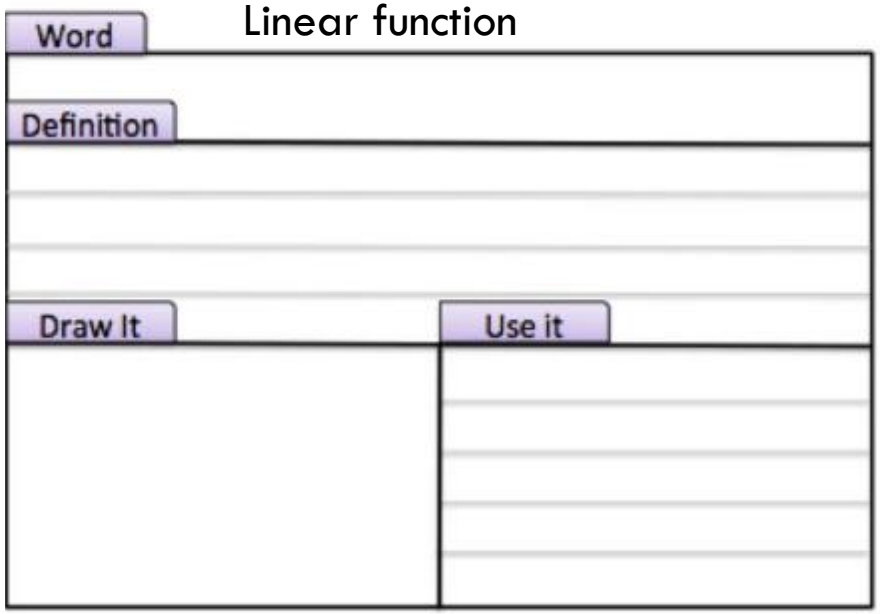
Polynomial function



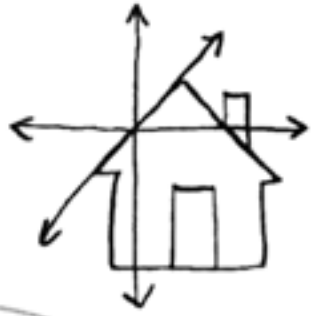
Linear function



My Definition

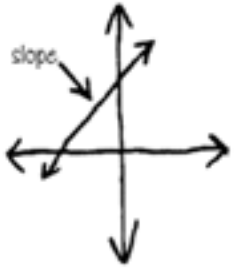
The slant of a line

Personal Association



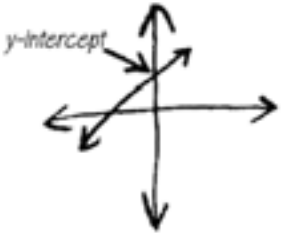
Slope

Example



Slope is 1. Up 1 right 1.

Nonexample



Slope is not the y-intercept.

Characteristics/Attributes

Rate of change

$$\frac{m = y_2 - y_1}{x_2 - x_1}$$

Rise over run

Positive – increasing to the right

Negative – decreasing to the right.

Teacher Definition

The ratio of the change in y compared to the change in x that expresses the rate of change.

Figure 3.9: Sample vocabulary organizer.

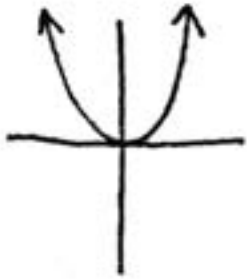
My Definition

Function that is not a constant or a line.

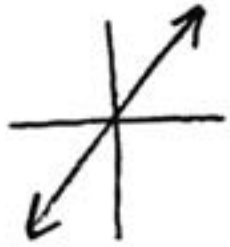
Personal Association

Quadratic Function

Example



Nonexample



Characteristics/Attributes

$ax^2 + bx + c = y$

$y = x^2$

Teacher Definition

An equation that can be $ax^2 + bx + c = y$ where a, b, and c are real numbers and $a \neq 0$.

Figure 5.9: Sample completed vocabulary organizer.

Name: _____

Vocabulary Sketches

Word: _____ **Definition:** _____

Sketch

_____

_____

Sentence: _____

_____

_____

Word: _____ **Definition:** _____

Sketch

_____

_____

Sentence: _____

_____

_____

Word: _____ **Definition:** _____

Sketch

_____

_____

Sentence: _____

_____

_____

Vocabulary Notebook

New Word and Illustration	Pronunciation	Definition in Your Own Words	Sentence and/or Connections

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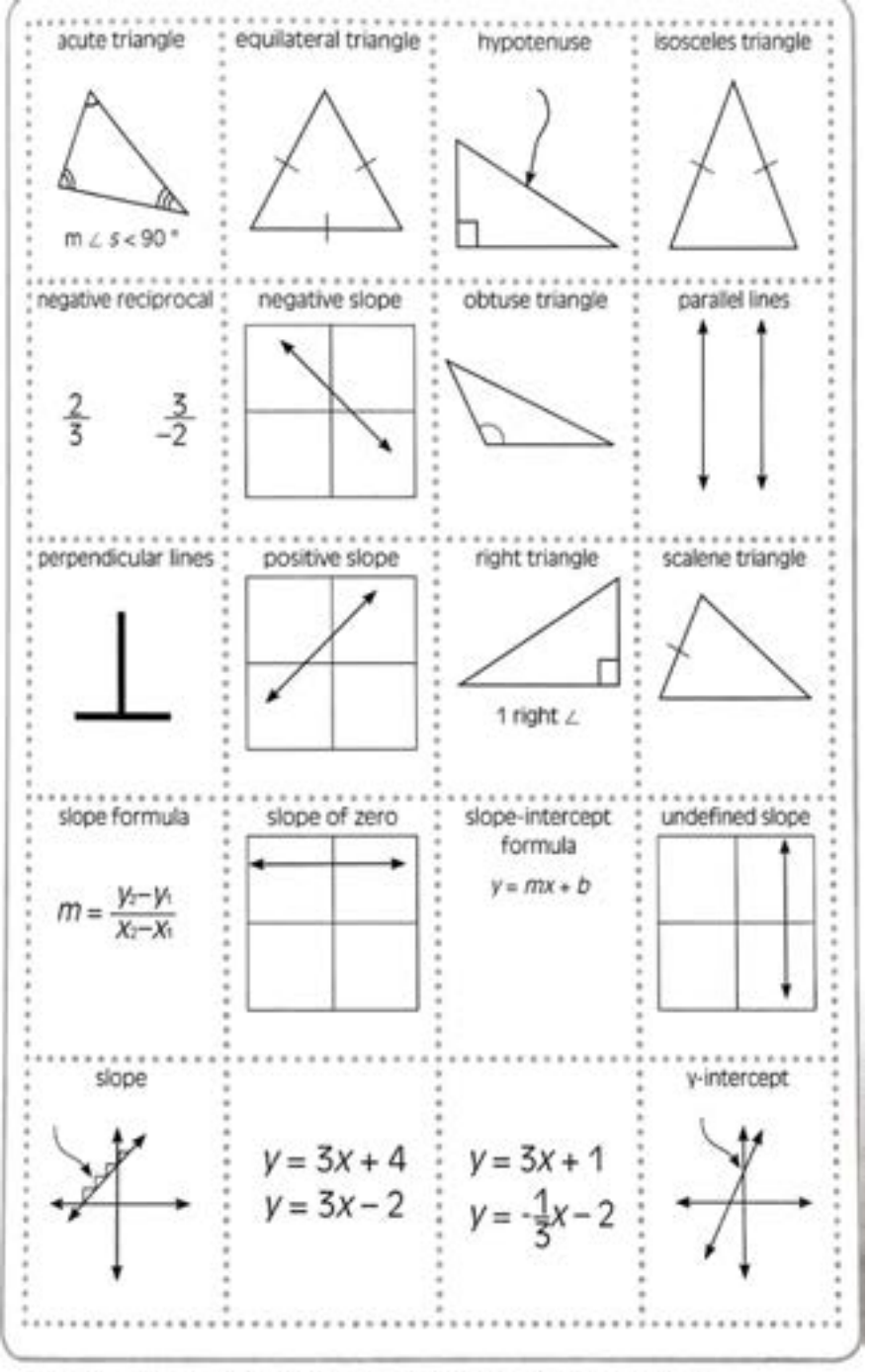
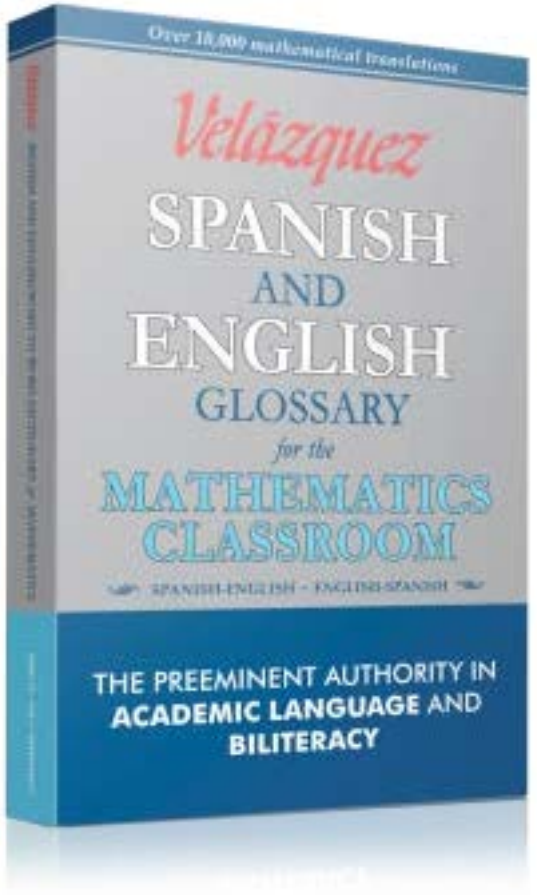


Figure 6.11: Sample word bank cards for an adapted lesson.

Word Bank Cards; Cognate Glossaries



https://velazquezpress.com/products/item/velazquez-spanish-and-english-glossary-mathematics-classroom

VOCABULARY SELF-RATING

Purpose This activity alerts students to the key words they will learn and helps them plan and monitor their learning. It helps students be aware of what they know, and take responsibility for what they need to learn. The teacher adjusts lessons based on a quick review of students' personal rating sheets.

Description Students rate their knowledge of key vocabulary words before and after the *investigate* phase of the lesson. A student's self-rating is personal; it may be shared with the teacher, but it is not graded. Students rate whether they know the word (K), do not know the word (DK), or are not sure (?) at three different points: before the lesson begins, after specific vocabulary instruction, and after instruction on mathematics content (at the end of the entire lesson).

Use

- *Introduce* phase: The teacher pronounces each word and students rate their knowledge level. This alerts students to words they need to learn. A quick survey of completed columns alerts the teacher to which words to emphasize.
- *Investigate* phase: Students rate the words again after vocabulary instruction. Students see their growth and the teacher sees which words need more attention during the content lesson.
- *Summarize* phase: Students rate the words once again and see their growth, while the teacher sees which concepts need further discussion and what content related to the words may not be sufficiently understood.

Example Students rate their knowledge of key words that are important in the day's lesson on division of whole numbers.

Vocabulary Self-rating			
Name:			
Lesson Topic:			Period:
K: I am sure I know it	**DK**: I am sure I don't know it		**?**: I'm not sure
Word (part of speech)	Before Lesson	After Vocabulary Discussion	After Lesson
Divisor (n.)			
Division (n.)			
Quotient (n.)			
Divided (v.)			

Lesson Framing Strategies

In this lesson we will learn about:

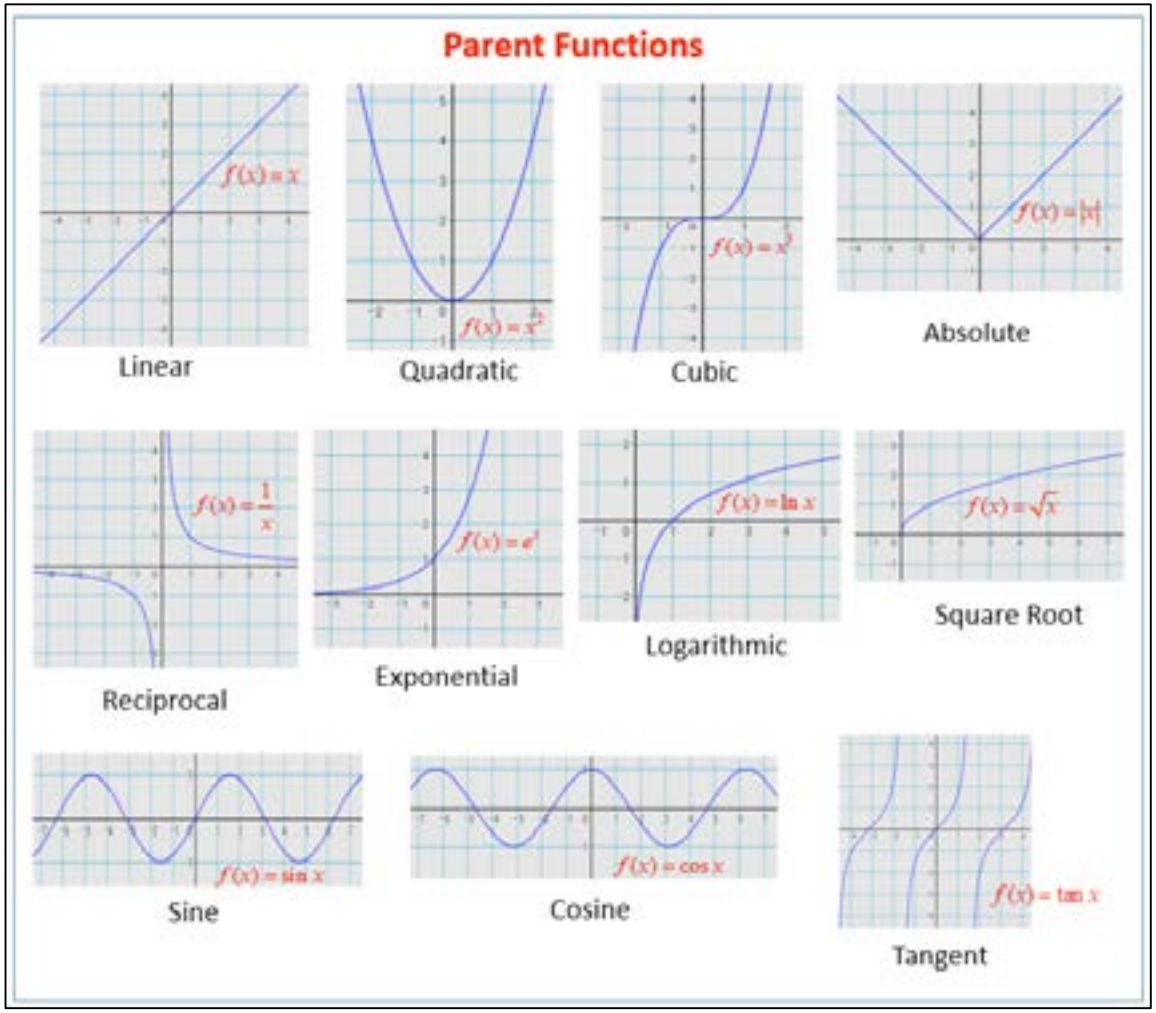
- Proofs
- Theorems about Angles and Lines
- How theorems can be used as facts in proofs of other theorems

Equation	**TERM & POLYNOMIAL FUNCTION**
	Constant function (Degree 0)
	Linear function (Degree 1)
	Quadratic function (Degree 2)
	Cubic function (Degree 3)

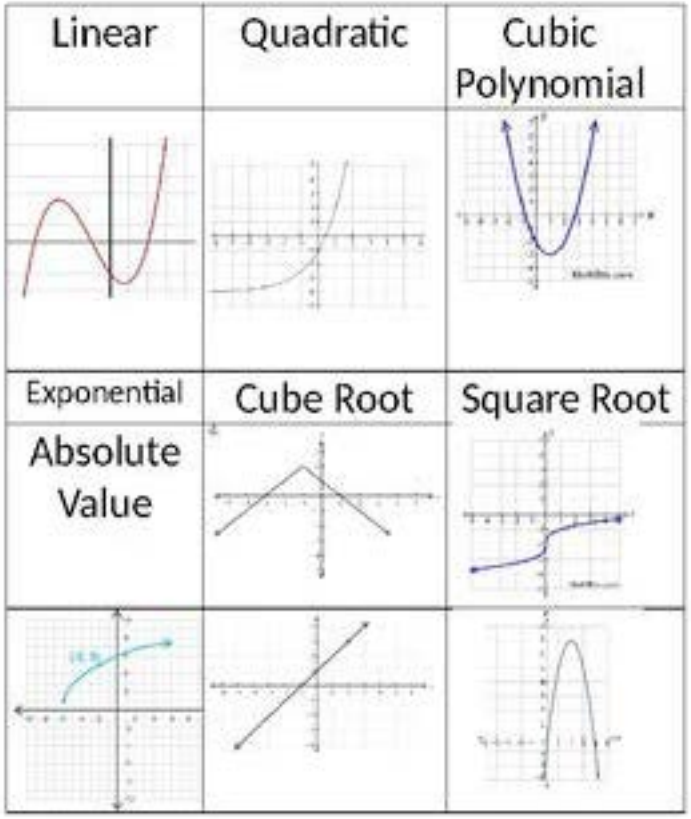
T-Chart with notations or drawings

Provide Concept Organizers to Complete as the Lesson Progresses

- Charts or Tables
- Diagrams
- (Partially Completed) Content Outlines
- T charts or Cornell Notes (2-column notes)
- Cumulative Summaries
- Daily Previews
- Focus Question—Cycle Back to Answer



Charts and Diagrams



PEARSON LANGUAGE CENTRAL for MATH

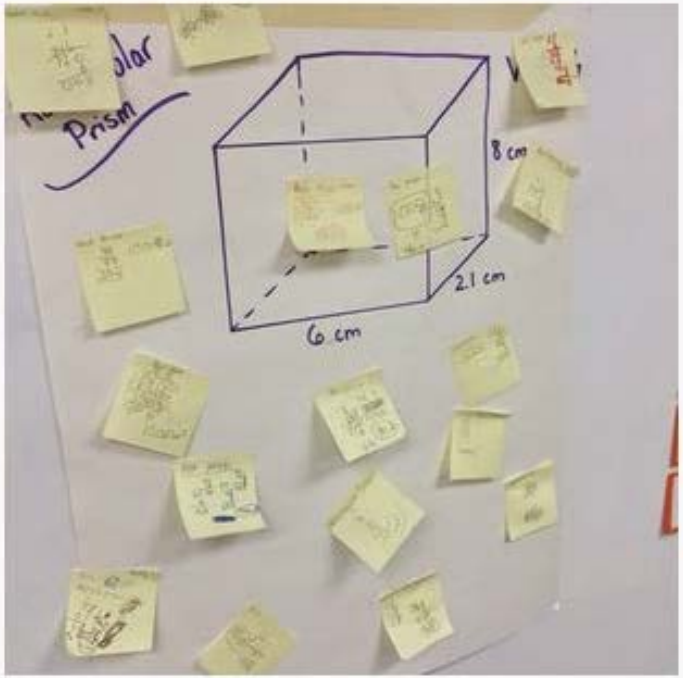
Research-based instructional design

5 principles for teaching ELLs

Interaction/Practice/Language Use Strategies

Post it-Pile It - A technique to help encourage and increase participation

When posing questions to a class mixed with ELL, students who do not feel confident in their language skills may hesitate to respond. One way to increase participation in your classroom and help to include ELL in the discussion is to use the 'Post-it-Pile-It' technique. The teacher gives post-it notes to each student (they can vary in size depending on the length of the answer), poses a question and then has every student answer the question on the post it, the students then pile their answers under the question on the flip chart or board. This technique also makes the student responses anonymous, helping to encourage confidence in participation.



Practicing Language While Deepening Content Learning

Language

- Help students use the core vocabulary you planned for the unit
- Make sure to extend students speaking ability: length and complexity
- Give sentence frames and starters

Activities that Allow Students to Deepen Content Knowledge while Using Language

- Show diagrams, equations that students have to explain to a partner
- Give charts, diagrams, visuals for students to complete (We do, then I do)
- Practice multiple choice items in quizzes and share responses; discuss why certain answers are the correct ones

Sentence Frames

Sentence frames. The following samples can be posted as a scaffold as students learn and practice their reasoning and oral participation skills.

- I think , because …….
- I predict , because ……..
- I claim ; my evidence is …….
- I agree with that …….
- My idea is similar/related to NAME's idea…….

Sentence Frames

- I learned/discovered/heard that ……..
- <Name> explained to me……..
- <Name> shared with me……..
- We decided/agreed that ……..
- Our group sees it differently, because …..
- We have different observations/results. Some of us found that……
- One group member thinks that ……..
- We had a different approach/idea/solution/answer .

FIGURE 4.5 Discussion Sentence Starters

Predicting
- I guess/predict/imagine that...
- Based on..., I infer that...
- I hypothesize that...

Asking for Clarification
- What do you mean?
- Will you explain that again?
- How did you find your answer?

Soliciting a Response
- What do you think?
- We haven't heard from you yet.
- Do you agree?
- What is your solution? How did you get it?

Affirming
- That's an interesting idea.
- I hadn't thought of that.
- I see what you mean.

Reporting a Partner's Idea
- ___ shared with me that...
- ___ pointed out to me that...
- ___ emphasized that...
- ___concluded that...

Disagreeing
- I don't agree with you because...
- I got a different answer than you.
- I see it another way. I think...

Expressing an Opinion
- I think/believe that...
- In my opinion...
- It seems to me that...
- Based on my experience, I think...

Paraphrasing
- So you are saying that...
- In other words, you think...
- What I hear you saying is...

Acknowledging Ideas
- My idea is similar to/related to ___'s idea.
- I agree with ___ that...
- My idea builds upon ___'s idea.

Holding the Floor
- As I was saying...
- If I could finish my thought...
- What I was trying to say was...

Reporting a Group's Idea
- We decided/agreed that...
- We concluded that...
- Our group sees it differently.
- We had a different approach.

Offering a Suggestion
- Maybe we could...
- What if we...
- Here's something we might try.

Source: Adapted from *Language Strategies for Active Classroom Participation* (June 2007) with permission from Kate Kinsella. The document can be accessed as LanguageClassDiscussion.doc at http://www.sccoe.org/depts/ell/kinsella.asp. This webpage also lists many other "open access" documents that Kate Kinsella presents in her workshops.

Cooperative Learning/TPTs

- Assign groups/pairs
- If a Group; Assign roles within the groups
- Assign the task
- Provide appropriate amount of scaffolding
- Intervene to ensure full participation
- Keep students on track

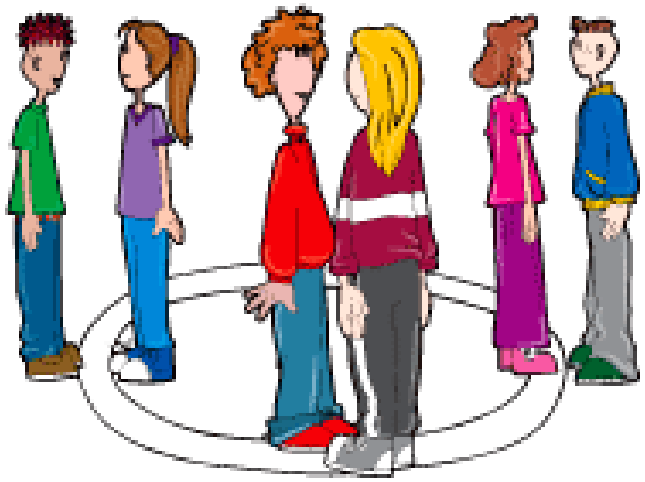
Speaking Activities

- Reporting back
- Inside-Outside Circle
- Numbered Heads
- Snowball Questions

Cooperative Structures

- Numbered Heads Together
  - Students huddle to make sure all can respond, a number is called, the student with that number responds.
    - Paired Heads Together: Students in pairs huddle to make sure they both can respond, an "A" or "B" is called, the student with that letter responds.

Plan an Inside-Outside Circle for Our Unit to Give Students Practice



Snowball Technique

The Snowball cooperative learning strategy is a dynamic activity that allows the students to move around the classroom making use of their social and mastery skills. In this activity half of the students receive the questions on a colored sheet of paper and the other half of the students receive the answers on a different colored sheet of paper.

The students who have the same colored sheet of paper line up side by side and an imaginary line is drawn between the two groups of students. Once everyone is in line the teacher will ask the students to wad up their piece of paper and throw it across the imaginary line to the other side. The students then pick up one "snowball" and collaborate with others to find either their part of the question or the answer.

For example if the students were working on their multiplication facts the question would be a math problem and the answer would be a number. The students would then go around the classroom communicating with the other students until they find their match. You could also use this strategy to focus on vocabulary, review for a test, or match history events to their dates.

Providing Feedback to Learners: What and How to Provide Language Correction

Rules in Providing Feedback:

- Don't just make incidental corrections; Show the student how English works or provide practice
- Include the whole class in the practice activity
- Use **Form-focused correction (FFI)**—show <u>one thing</u> you are noticing the students are doing (e.g. third person singular in present tense—leaving off the "s"—it go vs. it goes; not using have/has correctly); Keep on it till the production changes, help students self-edit for things you have taught them about English
- Choose pronunciation errors that disturb meaning (substraction instead of subtraction; *yust-ify* vs. justify)

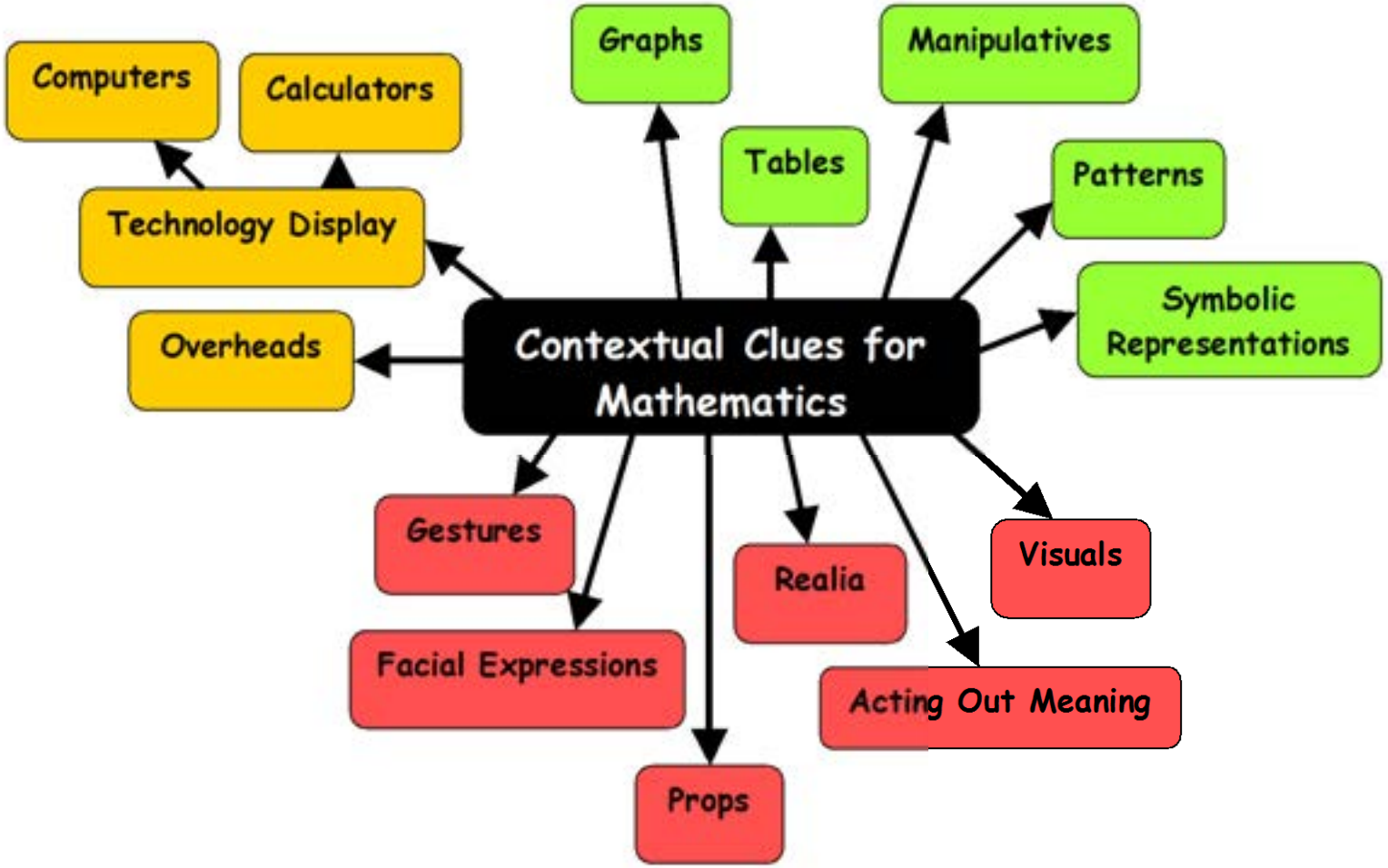
Resources for Math Teachers of ELLs

Description of English Learner Strategies

1	Predictable Routines and Signals	Reducing anxiety non-language based classroom management
2	Advanced Organizers	Informing students of the learning goals
3	Preview / Review	Building vocabulary and concepts to support understanding, summarizing, synthesizing
4	Academic Language Scaffolding	Supporting student use of language in academic settings (sentence frames)
5	Visual Scaffolding/Imaging	Providing language support through visual images (Multiple representations)
6	Vocabulary Development Word Walls, Dictionaries	Displaying and organizing words for easy access: Frayer and JAN models, student dictionaries
7	Communication Practices & Cooperative group work	Creating opportunities for verbal interaction about the mathematics Discussions: Pair-Share, small group, whole class
8	Modified teacher speech; Paraphrasing	Paraphrasing, repeat idea with correct vocabulary, adjust rate of speech, enunciate clearly (hundreds vs. hundredths)
9	Leveled Questions	Adjusting questioning strategies to the language and mathematics levels of students
10	Story Reenactment	Using the "Act it out" strategy for a context or problem
11	Realia Strategies	Connecting concept acquisition using real world objects
12	Manipulatives	Connecting concept acquisition using specially designed
13	Total Physical Response	Integrating movement into concept Acquisition
14	Modified Assessment	Less paper and pencil assessment

Effective Strategies for Teaching Mathematics Content to English Learners
CABE Conference
March, 2008

Judith E. Jacobs, Cal Poly Pomona
jejacobs@csupomona.edu



Modified from *Sheltered Instruction Across the Disciplines: Successful Teachers at Work*; Linda Sasser, Beth Winningham

Selected Research for ELD and SDAIE Strategies for Mathematics Instruction

	Strategy	Research
1	Predictable Routines and Signals	Even though your content will vary, follow a predictable routine and a stable schedule. Predictability in routine creates a sense of security for students who are experiencing a lot of change in their lives. (Peregoy & Boyle, 1997)
2	Advanced Organizers	The final strategy is the use of *modeling, graphic organizers, and visuals*. The use of a variety of visual aids, including pictures, diagrams, and charts, helps all students—and especially ELL students—easily recognize essential information and its relationship to supporting ideas. Visuals make both the language and the content more accessible to students. (Alliance, 2005, p.2)
3	Preview/ Review	Results indicate that not only did the students in the preview-review group score significantly higher than the control and concurrent translation groups, the concurrent translation group scored the lowest of all three groups and improved slightly one week after treatment. These findings demonstrate positive implications for the use of strategies which build background knowledge as a means of teaching second language vocabulary to English learners. (Ulanoff & Pucci, 1999, p.319)
4	Academic Language Scaffolding	Classroom instruction should support bilingual students engagement in conversations about mathematics that go beyond the translation of vocabulary and involve students in communicating about mathematical concepts. One of the goals of mathematics instruction for bilingual students should be to support all students, regardless of their proficiency in English, in participating in discussions about mathematical ideas. Teachers can move toward this goal by providing opportunities for bilingual students to participate in mathematical discussions and by learning to recognize the resources that bilingual students use to express mathematical ideas. (Moschkovich, 2002, p.208)

5	Visual Scaffolding/ Imaging	Learners acquire and store knowledge in two primary ways: linguistic (by reading or hearing lectures), and nonlinguistic (through visual imagery, kinesthetic or whole-body modes, and so forth). The more students use both systems of representing knowledge, the better they are able to think about and recall what they have learned. (Marzano, Pickering, & Pollock, 2001)
6	Vocabulary Development Word Walls, Dictionaries	Sometimes teachers focus so much on vocabulary and correct grammar that they neglect to teach students how to use the language they are learning. Doing math is no longer just a matter of listening to the teacher, doing computations correctly, and solving story problems. Students must be able to analyze, interpret, categorize, compare, describe, explain, demonstrate, present, and so forth. For any of these functions of language, students must know what words, phrases, and sentence structures to use, and how to use them. Functions can be effectively taught through modeling, followed by guided practice and independent practice in pairs or small groups. (Irujo, Nov/Dec 2007)
7	Communication Practices/ Cooperative group work	My review of the research showed that for ELL students working in groups, three factors mattered. First, students within the groups should not be homogeneous in their language ability. Students of different language ability need to interact in order to improve the group's mathematical communication. Next, the groups' composition needs to change periodically so that students do not become complacent with group work. Varying the groups' makeup also gives students the chance to gain insight from many different students' mathematical point of view. Finally, students need to learn how to participate in groups. ELL students have likely not participated in the type of group work most teachers may use. Rather than assume that they are adept at working in groups, and thus set them up for marginal success in your classroom, take time to teach ELL students how to work in groups. (Winsor, 2007, p.374)

8	Modified teacher speech	Although it is often recommended that teachers of ELLs use simple vocabulary and sentence structure in order to make their lessons comprehensible, academic vocabulary should never be simplified. Students need to master those terms in order to achieve on math proficiency tests. (Irujo, Nov/Dec 2007)
9	Leveled Questions	Leveled questions are used when teachers adapt the way they ask questions so that students can answer or respond to then according to their language acquisition stage. This may involve the teacher using gestures, visuals, or slowing the speech slightly while asking the question.
10	Story Reenactment	Learners acquire and store knowledge in two primary ways: linguistic (by reading or hearing lectures), and nonlinguistic (through visual imagery, kinesthetic or whole-body modes, and so forth). The more students use both systems of representing knowledge, the better they are able to think about and recall what they have learned (Marzano, Pickering, & Pollock, 2001).
11	Realia Strategies	One of the greatest problems English learners face in content area classes is reading the textbooks. Not only is the language academic, but it is usually very dry and dense, with few or no relevant illustrations, and presented in a linear rather than cyclical way. Embedding this language in a sensory context by using manipulatives, pictures, a few minutes of a film (without sound) and other types of realia (authentic objects and sources of information) can make language accessible and engaging for students. (Walqui, 2006, p.173)
12	Manipulatives	Sowell (1989) performed a meta-analysis of 60 studies to examine the effectiveness of various types of manipulatives with kindergarten through postsecondary students. Although these studies indicate that manipulatives can be effective, they suggest that manipulatives have not been used by many teachers. (Hartshorn & Boren, 1990)
13	Total Physical Response	Short TPR activities, used judiciously and integrated with other activities can be both highly motivating and linguistically purposeful. Many learners respond well to kinesthetic activities and they can genuinely serve as a memory aid. A lot of classroom warmers and games are based TPR principles. (Bowen)

14	Modified Assessment	One the basis of previous research, we believe that the following can be said with confidence at this time: 1. Translating test items from English into other languages does not appear to be an effective accommodation strategy when the students have studied the subject in a classroom where English is used. The language of assessment should match the students' primary language of instruction. 2. Some accommodations are more effective with certain student groups than with others, depending on background factors such as English reading proficiency and length of time in the United States. 3. The performance gap between English learners and other students has been narrowed by modifying the language of the test items to reduce the use of flow-frequency vocabulary and complex language structures that are incidental to the content knowledge being assessed. This accommodation is effective; it is also valid, because it does not appear to affect the performance of English-proficient students. 4. Customized dictionaries can be an effective and valid alternative to commercial dictionaries; they have been found to help English learners while not affecting the scores of English-proficient students. (Abedi et al., 2004, p. 17)

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Click to watch

A Model Algebra Lesson with ELLs

(6:30)

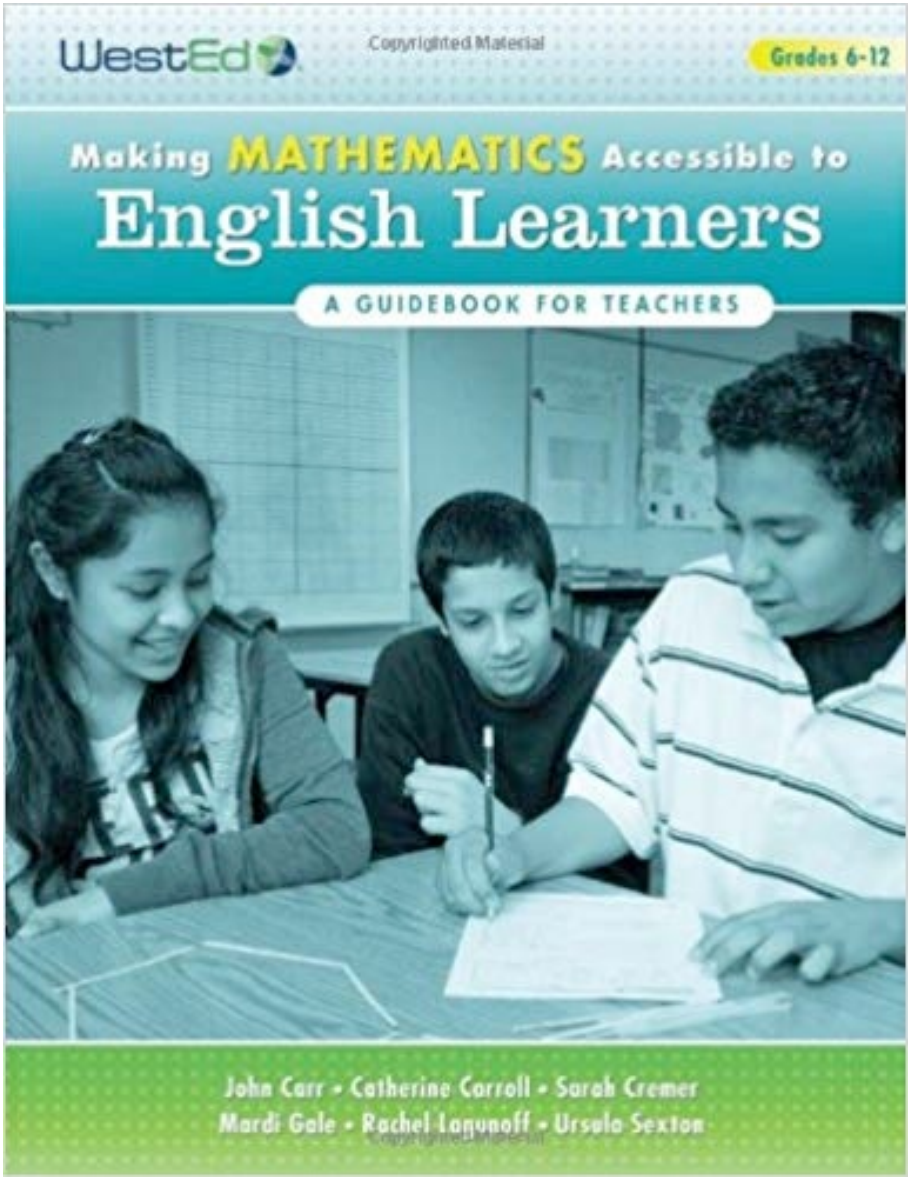
Focus: How does she get the student to understand the problem? (what scaffolds does she provide?)

How does she give the students practice with language?

Online PD for Teachers of ELLs

https://teachnkidslearn.com/u_course_cat/english-language-learners-ell/

ID	Course Name	Duration	Start Date
ELL Strand	Building Academic Language Skills and English Language Learners (ELL) through Dialogue, Discussion, and Discourse	Self-Paced	
Differentiated Instruction	Strategies for Assessment-Driven Differentiated Instruction, Grades K-12	Self-Paced	
Parent Involvement	Engaging Parents In Support of Student Learning, Grades K-12	Self-Paced	
Student Discourse	Academic Discourse for All Students, Grades 6-12	Self-Paced	
Literacy Strand	Complex Textual Reading Made Easy, Grades 6-12	Self-Paced	
Literacy Strand	Building Academic Vocabulary and Deep Comprehension, Grades K-5	Self-Paced	
Literacy Strand	Building Academic Vocabulary and Deep Comprehension, Grades 6-12	Self-Paced	
Deepening Learning	Differentiated Instruction Driven by Assessments (Foundations)	Self-Paced	
ELL Strand	Meeting ELL Students' Needs in Today's Classroom, Grades K-12	Self-Paced	
ELL Strand	Academic Achievement For English Language Learners (ELLs)	Self-Paced	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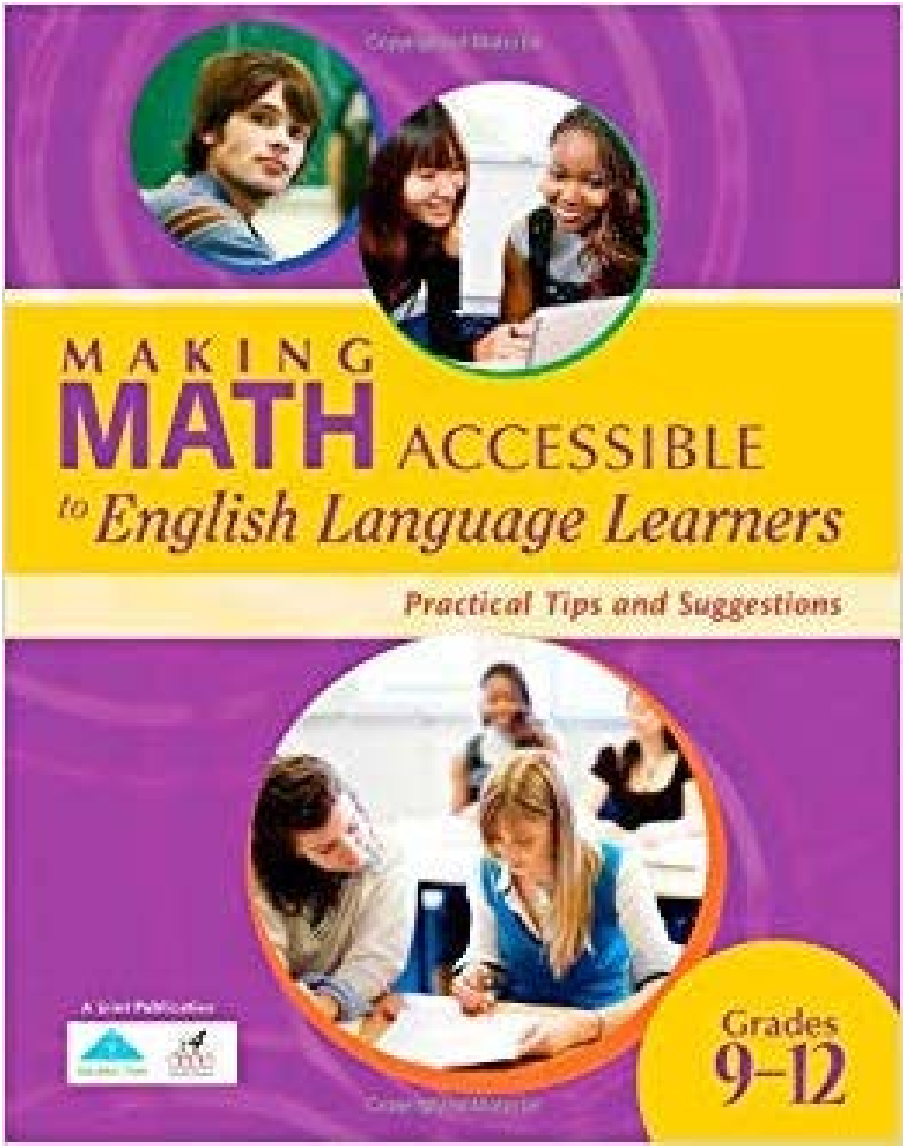
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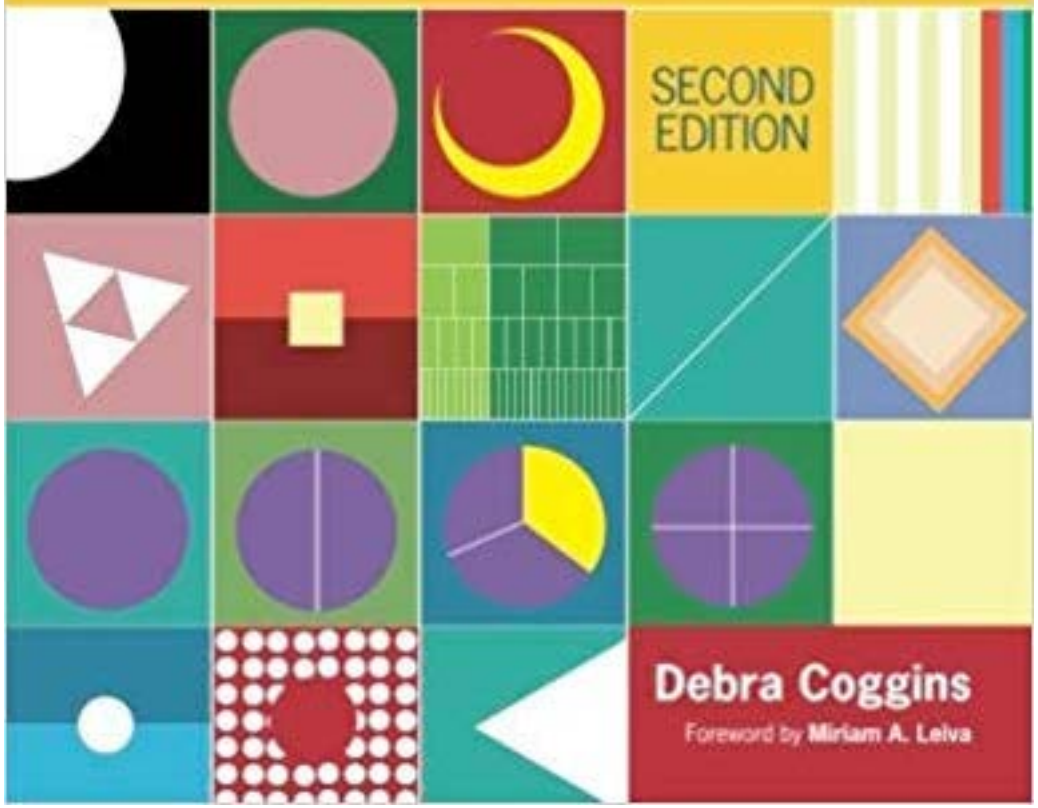

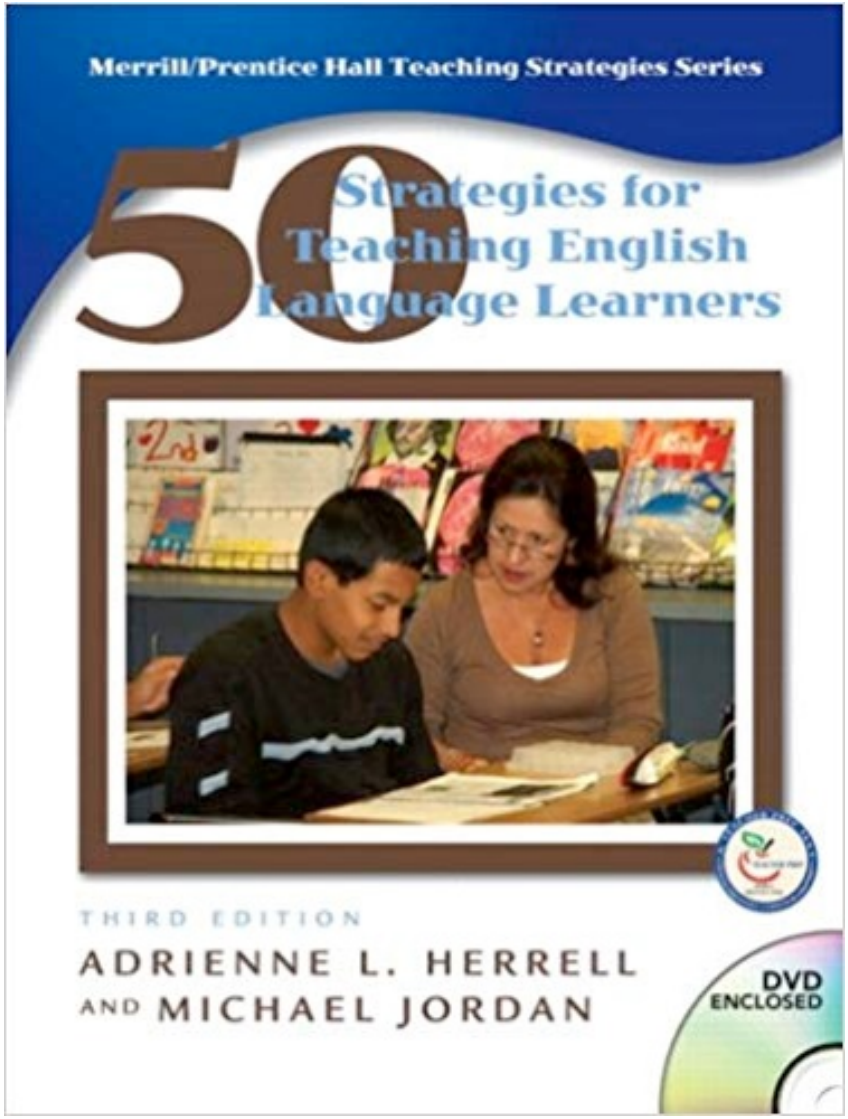
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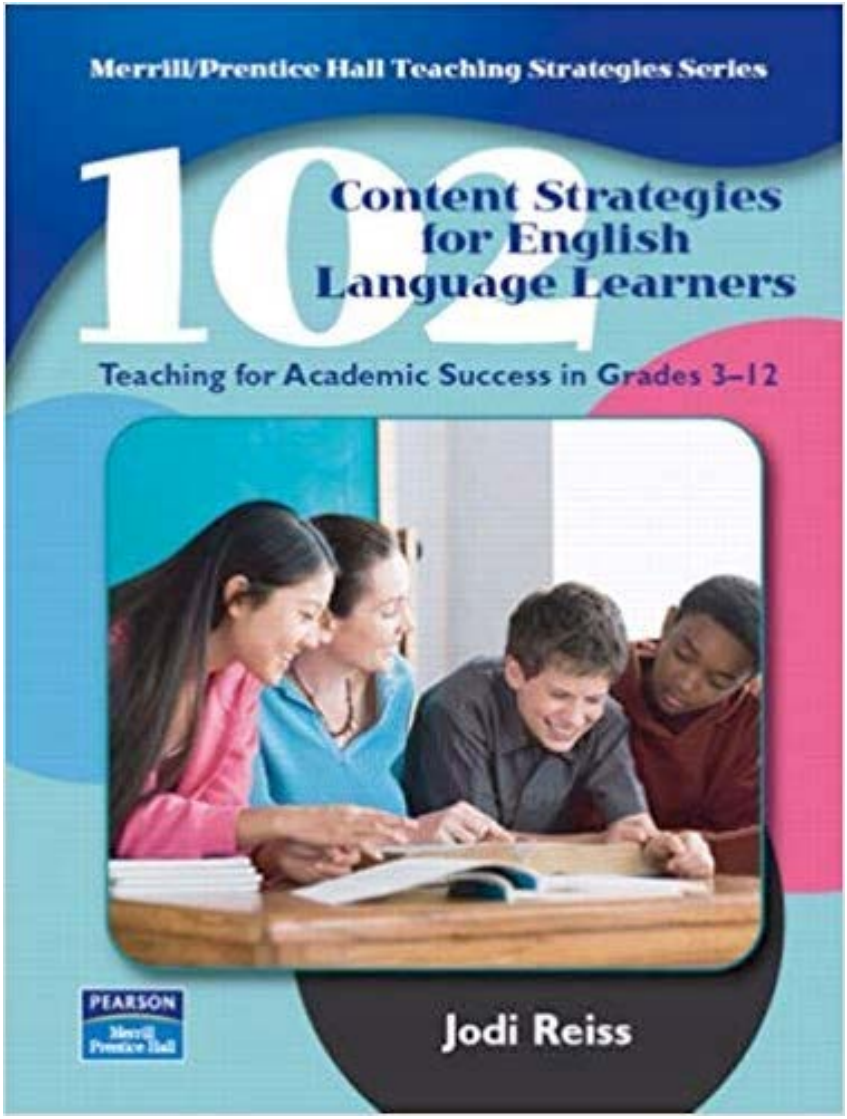
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Thank You!

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