NYU STEINHARDT New York State Statewide Language RBERN at NYU

Richmond Hill High School

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2= 02 × 0

y = mx + b

Math Best Practices: Scaffolding Content & Language for ELLs in the Math Classrooms

Monday, January 28, 2018





 $A = \pi r^{*}$

XJ

NYU STEINHARDTNew York State Statewide Language RBERN at NYU

Presenters: Mr. Archangelo Joseph Mrs. Marie-Alix Emmanuel

9:00 AM - 12:00 NOON

- Greetings
- Warmups
- PD rationale / Student Data
- Student Strengths & Challenges
- NYS Next Generation Standards
- Scaffolding Language
- Scaffolding Procedural Fluency
- Scaffolding Conceptual Understanding
- Scaffolding Problem Solving

@ Richmond Hill High School Professional Development

Mathematics Best Practices: Scaffolding Content & Language for ELLs in the Math Classrooms

Monday, January 28, 2018

AGENDA

LUNCH 12:00 PM to 12:50 PM

1:00 – 2:20 PM

- Practice/Group Works
- Presentations
- Reflections/Implications for Classrooms
- Theories on Best Practices
- Evaluation



The Magic Square

Place one of these number in each cell of the grid so that their sum equal 15, horizontally, vertically, or diagonally: 1, 2, 3, 4, 5, 6, 7, 8, 9. Do not repeat a number.



Rationale for this PD

1. It's the law that all teachers are teachers of ELLs (NYSED Blueprint for ELLs, April 2014), and must plan accordingly.

2. ELLs troubling academic achievement, including in mathematics, and graduate rate are significant factors to drop-out rate.

3. Students, especially ELLs, generally struggle on constructedresponse or open-ended math questions requiring a great deal of expressive language.

4. The upcoming Next Generation Standards promote language (including native tongue) and content knowledge, and culturally-based activities





1. All teachers are teachers of English Language Learners, and need to plan accordingly.

2. All school boards and district/school leaders are responsible for ensuring that the academic, linguistic, social, and emotional needs of ELLs are addressed.

3. Districts and schools engage all English Language Learners in instruction that is grade appropriate, academically rigorous, and aligned with the New York State Prekindergarten Foundation for the Common Core and P- 12 Common Core Learning Standards.

4. Districts and schools recognize that bilingualism and biliteracy are assets, and provide opportunities for all students to earn a Seal of Biliteracy upon obtaining a high school diploma by providing all students with opportunities to *develop academic language and content knowledge and English and in the home language.*

Combined Former and Current ELL Graduation Rate Remains Steady



New York State

Next Generation Mathematics Learning Standards

OVERVIEW

Next Gen Goals for High School

- Revised Common Core Learning Standards
- ELLs/MLLs and SWD students in mind
- Preparing lifelong learners and thinkers
- Exploration of some concepts & mastery of other topics
- Focal content and skills
- 18 shared standards between Algebra I and Algebra II
- Consolidation of some standards for coherence
- Integrated language and content instruction to support language development through language-focused scaffolds and ENL continuum
- Balance the need for conceptual understanding, skills and applications
- <u>Universal Design for Learning (UDL): Multiple means of engagement,</u> representing and expressing
- Instruction that is culturally and linguistically appropriate for all diverse learners, including those with Individualized Education Programs (IEP).
- Culturally & linguistically responsive quality/rigorous instruction and materials
- Glossary of verbs associated with Next Gen

http://www.nysed.gov/bilingual-ed.

Mathematical Practices shared by both

Common Core & Next Generation

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning

Algebra I

- Solving equations and systems of equations.
- Linear equations: Exploring, creating, reasoning, writing, interpreting, and translating, conjecturing as related to problem solutions.
- Polynomials & specific terminology
- Understanding analogy between operations with and integers
- Function notation, domain & range
- Interpreting arithmetic sequences (patterns) as linear functions
- Building models of relationships between two quantities
- Multiple representations (e.g., texts, graphs & tables)
- Discerning structure in polynomial expressions
- Creating and solving equations involving quadratic and cubic expressions.
- Comparing characteristics of quadratic & linear & exponential functions
- Interpreting the structure of various forms of quadratic expressions.
- Identifying the real solutions of a quadratic equation as the zeros of a related quadratic function
- Selecting model phenomena using the modeling cycle
- Students display and interpret graphical representations of data,
- Using knowledge of context to justify choice of linear model

Algebra II

- Building on their work with linear, quadratic, and exponential functions in Algebra I
- Discussing polynomial, rational, radical, and trigonometric functions.
- Working with expressions that define the functions
- Expanding and honing abilities to model situations and to solve equations, including solving quadratic equations
- Solving exponential equations using the properties of logarithms.
- Draw on analogies between polynomial arithmetic and base-ten computation,
- Focusing on properties of operations, particularly the distributive property.
- Connecting the structure inherent in multi-digit whole number multiplication with multiplication of polynomials
- Connecting division of polynomials with long division of integers
- Identifying zeros of polynomials, including complex zeros of quadratic polynomials.
- Reasoning repeatedly to they make connections between zeros of polynomials and solutions of polynomial equations.
- Analyzing the key features of a graph or table of a polynomial function and relate those features back to the two quantities the function is modeling.
- Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry
- Extending trigonometric functions to all (or most) real numbers.
- Reinforcing understanding of these functions, students begin building fluency with the values of sine, cosine, and tangent at $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, etc

Algebra II (cont'd)

Making sense of periodic phenomena while modeling with trigonometric functions. Synthesizing and generalizing what has been learned about a variety of function families. Extending work with exponential functions to include solving exponential equations with logarithms as well as understanding the inverse relationship between exponential and logarithmic functions. Exploring (with appropriate tools) the effects of transformations on graphs of diverse functions, including functions arising in an application. Identifying appropriate types of functions to model a situation. Adjusting parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. With repeated opportunities in working through the modeling cycle, acquire the insight that the same mathematical or statistical structure can sometimes model seemingly different situations. Seeing how the visual displays and summary statistics learned in earlier grades relate to different types of data and to probability distributions.

Identifying different ways of collecting data, including sample surveys, observational studies, and experiments. Using simulation, randomization, and careful design to make inferences, justify conclusions, and critique statistical claims. Creating theoretical and experimental probability models following the modeling cycle. Computing and interpreting probabilities from those models for compound events, attending to mutually exclusive events, independent events, and conditional probability.

Geometry

- Formalize geometry experiences from elementary and middle school
- Using more precise definitions to establish the validity of geometric conjectures through deduction, proof, or mathematical arguments.
- Understanding attributes and relationships of two- and threedimensional geometric shapes in diverse contexts.
- Understanding and applying the Pythagorean Theorem in many mathematical situations.
- Informal explanations of circumference, area, and volume formulas.
- Reasoning abstractly and quantitatively to model problems using volume formulas.
- Proving and applying basic theorems about circles and study relationships among segments on chords, secants, and tangents as an application of similarity.
- Discussing congruence, similarity, and symmetry, geometric transformations (e.g., translations, rotations, reflections, ...)
- Applying theorems about triangles, quadrilaterals, and other geometric figures.
- Defining sine, cosine, and tangent for acute angles on right triangles

The 4 Pillars of 21st Century Math





Language

Ultimate Goals of this PD

To discuss and engage in best math practices, i.e., connections with real life, through various scaffolding strategies and ample practice.

- 1. Scaffolding the language of mathematics
- 2. Scaffolding skills (procedural fluency)
- 3. Scaffolding conceptual understanding
- 4. Scaffolding problem solving strategies (Socratic Method)

PM

5. Group Activities: Solving real-life problems Sharing & debating solution strategies
6. Reflecting on/discussing implications for one's classroom, henceforth.

Bloom Taxonon	ny Depth of Knowledge	Universal Design
	 design 	Multiple Means of:
Evaluation	LEVEL 4 Prove	Engagement
Synthesis	Ext. Think Synthesize	defending viewpointsseeking clarifications
Synthesis	 critique 	 posing probing questions making commonts
Analysis	 investigate LEVEL 3 Strat Think problem solve 	 making comments conjecturing/hypothesizing
Application	 identify patterns LEVEL 2 Sk/Cncpt. classify 	 Representation Written reports / texts Graphic organizers Dramatization Film/video
Comprehension	- classily	Expression
Knowledge	LEVEL 1 ■ memorize Recall ■ recall	 Oral presentation /Debate Play Persuasive essay

Scaffolding Manipulatives Continuum

From most Concrete to most Abstract



MOST COMPREHENSIBLE INPUT

LEAST COMPREHENSIBLE INPUT In quest for i+1 (Krashen)

SCAFFOLDING LANGUAGE

Vocabulary

- Written word problems (as opposed to mere computations) present a unique challenge to ELL students and teachers alike.
- Modify the linguistic complexity of language and rephrase math problems.
- Guide students to cross out the unnecessary vocabulary in word problems.
- Build knowledge from real world examples.
- Use manipulatives purposefully.

Explicit Teaching of

Academic Vocabulary

- Demonstrate that vocabulary can have multiple meanings.
- Encourage students to offer bilingual support to each other.
- Provide visual cues, graphic representations, gestures, realia, and pictures.
- Identify key phrases or new vocabulary to pre-teach.
- VVWA, Riddles, glossary-word puzzles (AJ)

http://www.colorincolorado.org/article/math-instruction-english-language-learners





Word Etymology

Volume = volumen (latin for amount/size of roll/manuscript) Capacity = capere (latin for to take; capacitem = breadth, capability of holding much). Equation = equation (Latin for an equalizing) Angle = angulus (Latin for sharp bend. Also: ankle) Polygon = polus (Greek for many) + gonia (angle, corner) Acute = acus (Latin for needle, sharp) Diagonal = dia (Latin for to pass through or join) + gonus (angle) Diameter = diametros; dia (Greek for pass through or join) + metron (Measure) Exponent = exos (Latin for out of) + ponere (to place) Fraction = fractio (Latin for breaking); frangere, to break **Isosceles** = iso (Greek for the same) + skelos (legs) Polyhedron = poli (Greek for many) + hedros (face) Mono/bi/trinomial = mono =1; bi = 2; tri = 3 + nomos = Greek for portion, part **Geometry** = geo (Greek for Earth) + metria (measure) Slope = sleubh (Latin for slip) Hypotenuse = hypo (Greek for under) + tein (stretch) **Congruent** = con (Latin for together) + ruere (fall); congruere (to come together) Chord = chorde (Greek for string) Circle = circus (Latin for circular race track)

Kilo = 1,000; hecto = 100; deca =10; deci = 1/10; centi = 1/100; milli = 1/1000

Types of Academic Words

One-meaning Words

exponent total equation coefficient trapezoid hypotenuse Multiple-meaning Words

table dividend root power

terms

domain volume gross bank odd

Math Glossary Puzzle

Match the words with their definitions on the next page. Then locate then on the grid

TOTAL OPT ROOT TOOL INTEGER NAME CHORD REPORT ALL LOCUS PERCENT NO PROPORTION ADD **EPICENTER** REST PRACTICAL SUM TOP NONAGON FACTOR LAW ORIGINAL

What math task (s) can you develop from this letter grid?

Ρ	R	Ο	Ρ	0	R	т	Т	Ο	N
E	E	R	Ρ	Ρ	ο	S	U	М	ο
R	S	I	R	Т	ο	т	Α	L	N
С	т	G	A	S	т	ο	0	L	A
Е	Ρ	I	С	Е	N	т	Е	R	G
N	ı	N	т	E	G	E	R	E	0
т	В	A	I	N	Ο	т	0	Ρ	N
A	L	L	С	н	Ο	R	D	Ο	A
D	A	F	A	С	т	0	R	R	М
D	w	Y	L	0	С	U	S	т	Е

Definitions

ALL	Meaning "everything"
ROOT	Base of a power. May also mean part of a plant.
TOTAL	Synonym for sum

To make your own, go to: Puzzlemaker.discoveryeducation.com

Language Production

- Have students translate symbols into words, and write the sentence out.
- Create a "sentence frame" and post it on the board.
 - Have students share problem-solving strategies.
 - Allow students to discuss how they are thinking about math.
 - Incorporate writing activities like math journals and reports.

Challenge students to create their own math problems.

Math Discourse Features (Dr. Cloud)

- Conceptually packed.
- High-density of important words.
- Require up-and-down as well as left-to-right eye movements.
- Require reading-rate adjustment.
- Require multiple readings.
- Use numerous symbolic devices.
- Contain a great deal of technical language with precise meaning.

Foundational Math Symbols

X

+

% < 2 > + # < ~ Ø # ¢ Æ U E \cap { } () ß α \Rightarrow

•

 $\subseteq \Delta \lambda \pi \Sigma \theta \sim \infty$

Syntax within the Math Register

The syntax / sentence structure of math can be troublesome for ELLs.

Example: Solve: 2x + 3 subtracted from 5x - 1.

Solution A 2x + 3 - 5x - 1 2x - 5x - 1 + 3-3x + 2 Solution B 5x - 1 - (2x + 3) = 5x - 1 - 2x - 3 = 5x - 2x - 3 - 1 = 3x - 4

Visual-Verbal-Word Association (VVWA)



I have ...

Who has...?





of Teaching & Learning

The Socratic Method

1. Hypothesis / Claim / Statement

- 2. Clarification of the Hypothesis
- 3. Experimentation / Proof
- 4. Validation / Adjustment / Rejection of hypothesis

Graphic Organizers & Other Scaffolds



Frayer Model







What I know	What I want to know	What I have learned
PRIOR-KNOWLEDGE	GOAL LESSON OBJECTIVE	OUTCOME
I know that capacity is the total amount that can be contained in a 3-D object.	How is capacity different from volume?	I have learned that capacity is the amount of substance that a 3- D (solid) can contain based of measures from inside, whereas the volume is the space that the solid

occupies based on

measures from

outside.



Capacity

Volume



Anticipatory Guide

					Not		
Mathematical Statements					sure	Disagree	
1. A square has the properties of both rectangle and rhombus.							
2. Any quadrilateral with 4 congruent sides is a square.						Ν	
3. The diagonals of a rhombus are perpendicular.							
4. The sum of the measures of all angles in a triangle is 108°.						Ν	
5. The quotient of the sum of a an	d b, and their difference can be wri	tten as: $\frac{a+b}{a-b}$	Y				
6. "Three subtracted from five" ca	n be written as "3 – 5".					N	
7. The difference between the square of 3 and the double of 3 is 3.							
8. All improper fractions are greater than one.							
9. In the linear equation defined as $y = mx + 3$, x is the slope.						Ν	
10. Decimal 0.04785 is greater than decimal 0.4.						Ν	
11. The diagonals of a rectangle an	re always perpendicular.					N	
12. Constant proportionality mean	ns slope (m) equation $y = mx + b$.		Y				

Math Riddles

1. I am a polyhedron 8 vertices, 12 edges, and 6 faces. Who am I? **RECTANGULAR PRISM**

2. I am a space figure with no vertex just like the globe. Who am I? SPHERE

3. I am a number that can be represented by the equation n = 40 - 10. Who am I? 30

4. I am an even number. If you add six to me, and then subtract two, the result will be eighteen. Who am I? 14

5. I belong to a class of numbers having only two factors. Who am I? PRIME NUMBER

6. I am a composite and even number that is worth 25% of 60. Who am I? 15

7. I am an equation or function whose graph is a parabola. Who am I? **QUADRATIC EQUATION/FUNCTION**

8. I am the theorem associated with the famous formula: $c^2 = a^2 + b^2$. Who am I? **PYTHAGOREAN THEOREM**

9. I am formed when swapping the numerator and the denominator of a fraction. Who am I? RECIPROCAL

10. We are two lines formed by the graphing of two equations with the same slope. Who are we? PARALLEL LINES

11. I am the inverse operation to division. Who am I? MULTIPLICATION

12. We are two lines formed by two equations, the slope of one is the negative reciprocal of the other.? Who are we?

PERPENDICULAR LINES


Pam drives on a bumpy road to Antonio's Fruit Store.



Pam feeds the meter to park her car.



Pam looks for apples to make apple sauce. She purchases six green apples for \$1.80 and gives a \$5 bill to Cashier.



The apple sauce is ready for dessert. Yammy!



The family helps Pam peel off the apples.



Pam drives back home with the apples.

<u>Narrator</u>: Ladies and gentlemen, please welcome Jeanne and Mary. Jeanne is a young lady who loves mango juice. Mary is a peddler who wants to sell as many mangoes as possible. This morning, out goes Jeanne in search of her favorite fruit. As she crosses a street corner, a soft voice breaks the silence in her mind.

- Mary: My dear beautiful lady, would you come and check my mangoes out?
- Jeanne: Ok. No problem. Indeed, I do need some mangoes to make some juice.
- Mary: That's a good idea. Today they're fresh and delicious.
- Jeanne : How do you sell these *Francique* mangoes?
- Mary: Fifty cents each!

Planet Mice

- Jeanne : Ok. No problem. I'm taking ten.
- Narrator: Freeze! How much does Jeanne owe for the Francis mangoes?
- Mary : Mmm! Let's see... \$____
- Jeanne : Ok! I also need some *Cinnamon* mangoes.
- Mary: That's a good idea. Today they're sweet and juicy.
- Jeanne : Is it the same price?
- Mary: Nope! These are two for one dollar.
- Jeanne : Ok. No problem. I'm taking three.
- Narrator: Freeze! How much does Jeanne owe for these Cinnamon mangoe
- Mary : Mmm! Let's see... \$____
- Narrator: Freeze! How much does Jeanne owe in all?
- Mary : Mmm! Let's see... \$_
- Jeanne :Ok. No problem. Here's a crispy five-dollar bill. Keep the change!Narrator:Freeze! How much change (if any) has Jeanne left for Mary? Explain.Mary :Mmm! Let's see... \$_____



How can you use supermarket flyers to design meaningful activities for ELLs?

- Proportion / percent
- Proportional linear equation/function
- Non-proportional linear equation





Math Journals

Learning Logs

Scaffolding Skills & Conceptual Understanding



Metric System Prefixes

kilo = 1,000hecto = 100deca = 10Standard = 1 (meter, gram, liter) deci = $.1 = \frac{1}{10}$ centi = .01 = $\frac{10}{100}$ milli = .001 = $\frac{1}{1000}$

The System Metric (Table for Length)

km Thousands	hm Hundreds	dam _{Tens}	m Ones	dm Tenths	CM Hundredths	mm Thousandths
1,000	100	10	1	.1	.01	.001
$\frac{1000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
5	0	0	0			

5 km = 5,000 m

Note: 1 km = 1,000 m

System Metric Table for Area



$5 \text{ km}^2 = 5,000,000 \text{ m}^2$

Note: 1 km² = 1,000,000 m²

System Metric Table for Volume/Capacity



Note: 1 km³ = 1,000,000,000 m³

The Meaning of Division of Fractions

<u>Why</u> $3 \div \frac{1}{2} = 6?$

What does it means to divide 3 by 1/2?

What real-life situation would require the division above?

Concrete Division for Profound Understanding

1/2

1/2

 $\frac{1/2}{1}$ $\frac{1/2}{2}$

6 halves mango

1/2

1/2

6

Why multiply by reciprocal

when dividing fractions?

2

1

There are too many fraction bars. The numerator of the main fraction is ½ and the denominator is ¼. The main fraction bar is the red bar. Let's neutralize the numerator ¼ via multiplication by reciprocal. $\frac{\frac{1}{2} \times \frac{4}{1}}{\frac{1}{4} \times \frac{4}{1}} = \frac{1}{2} \times \frac{4}{1}$

Hence, the denominator $\frac{1}{4} \times \frac{4}{1}$ has been neutralized (or becoming equal 1).

The entire division has been reduced to a multiplication of 1/2 by 4.

The sample principle applies in algebra.

 $\frac{a^2 - b^2}{a^2 + 2ab + b^2} =$

Reminder: $a^{2} - b^{2} = (a+b)(a-b)$ $a^{2} + 2ab + b^{2} = (a+b)(a+b)$

 $\Rightarrow b \div \frac{(a+b)(a-b)}{(a+b)(a+b)} =$

 $b \times \frac{(a+b)(a+b)}{(a+b)(a-b)} = b \times \frac{(a+b)}{(a-b)} =$ b(a+b) a-b

Why $10^{\circ} = 1?$

So, $10^0 = 1$

Multiplication of powers: $10^3 \times 10^2 = 10^{(3+2)} = 10^5 = 10,000$

Division of powers: $10^3 \div 10^2 = 10^{(3-2)}$ $10^3 \div 10^2 = 10^1 = 10^1$

The same principle applies to: $10^3 \div 10^3 = 10^{(3-3)}$ $10^3 \div 10^3 = 10^0$ $10^3 \div 10^3 = 1$

Why the sum of the measures of all angles inside a triangle equals 180°?



<u>Why</u> π = 3.14?

	Circumference	Diameter	Ratio Value
\bigcirc			
\bigcirc			

<u>Why</u> Area of a Circle = πr^2 ?





Pythagorean Theorem



Why $c^2 = a^2 + b^2$?

Scaffolding Problem Solving

Socratic Method for Teaching & Learning

- 1. Declaration or Hypothesis
- 2. Clarification of Hypothesis
- 3. Experimentation / Demonstration / Proof
- 4. Validation / Modification / Rejection of Hypothesis

Taxi Ride



Facts (Taxi Fare)

- Initial fee = \$3
- 1/2 mile = \$1.50
- Distance = 16 miles
- Tip = \$5

Questions

Change out of \$50 bill?



(Application of the Socratic Method)

Construct three different 3-D shapes from identical sheets. Hypothesize on their capacities. Which has the greatest capacity. Prove it mathematically (using formulae) and empirically (using beans).



Contrasting Capacities

 Examine carefully your two differing solids.

- 2. Identify their nature and characteristics and predict which one has a greater capacity.
- 3. Use ruler, paper and pencil to (theoretically) determine and compare /contrast their capacities.
- 4. Use beans/corn meal concretely justify your findings.



Relations Equations Functions

Patterns

Multiple means of engagement
Multiple means of expression
Multiple means of representation

EQUATION = BALANCE







Written Text

Jane pays a one-dollar parking fee next to the neighborhood fruit store. She goes insides but purchases no apple, thinking the fruits are too expensive. Consequently, she returns home. Then she goes back again to the same store and, this time, purchases two apples, and her total expense, including parking, is five dollars. She returns two other times and grabs respectively three apples and four apples.

Color Tiles Picture Display From Patterns to Equations/Functions



Color Tiles Display of Realia

From Patterns to Equations/Functions

Realia (real, tangible apples/tiles)

SEE PLASTIC BAGS





Input-Output Table





Cartesian Diagram


Set of Ordered Pairs

$\mathsf{R} = \{(0,1), (2,5), (3,7), (4,9)\}$

Graph on Cartesian Plane



Set-builder Notation

 $R = \{(x,y) | x \in N, y \in N \text{ and } y = 2x+1 \}$

The Phone Problem

The Variables or Coordinates

abscissa Independent Variable Minutes

 \mathcal{X}

ordinate Dependent Variable Dollars

Plan A vs. Plan B

Facts:

<u>Plan A</u>

- \$6 fee
- \$0.50 per minute

<u>Plan B</u>

\$0 fee
\$2.00 per minute

Question:

Investigate which is the better plan. Explain.
 Use multiple strategies

2. For how many minutes the cost would be the same regardless of the plan. Explain. Use multiple strategies

Ex	be	cta	ati	0	ns
				-	

Organic-way Mathematics Problem Solving

Minutes <i>x</i>	Plan A Dollars y = 0.5x + 6	Plan B Dollars y = 2x				
0	6.00	0.00				
1	6.50	2.00				
2	7.00	4.00				
3	7.50	6.00				
4	8.00	8.00				
5	8.50	10.00				
6	9.00	12.00				
7	9.50	14.00				
8	10.00	16.00				

Double-Entry Table

Extending Thinking (Depth of Knowledge (DOK)

Into Geometric

- Area
- Fraction/Percentage
- Pythagorean Theorem
- Distance between two pints
- Transformations
- Etc.







Theoretical probability vs. Experimental Probability

Creating theoretical and experimental probability models following the modeling cycle. Computing and interpreting probabilities from those models for compound events, attending to mutually exclusive events, independent events, and conditional probability.

Probability

Sum of the numbers on the two dice

Investigation

Theoretical Probability

Experimental Probability

+	1	2	3	4	5	6	•••	+	1	2	3	4	5	6
1								1						
2								2						
3							•••_ 1	3						
4								4						
5								5						
6								6						
Find	lings:							E:	dima					

Findings:

PM GROUP ACTIVITIES



Gallon A

Full 5-gallon bottle to be emptied in 20

minutes.

Question:

Emptied 5-gallon bottle taking in 1/4 gallon per minute.

PÜR

Better

Determine when both containers will have the same amount of water. Use multiple strategies, including graphing.

Problem 2 Red Sweater on Sale

Facts:

Original Price: \$25 Everyday (x) price change according to equation

 $y = x^2 - 10x + 25$

Student has already \$7 in piggy bank, at the same time begins to receive \$1 daily from Granny.



Determine when (the first time) sweater can be purchased? Use multiple means, including graphing. Explain critical points on the parabola and the line.





Organic-way Mathematics: The Essence of Problem Solving Copyright © 2012 Problem 3

Tank Gaz

- After traveling a distance of 90 miles, a truck has 40 gallons left in its gas tank. But only 10 gallons have remained upon cruising a distance of 360 miles.
- 1. Generate an equation to show the relationship between the number of miles run (independent variable) the amount of gas left (dependent variable) in the tank.
- 2. Explain the type of equation and all its parts.
- 3. Indicate when (for what distance) the tank will be totally empty.
- 4. At departure, how much gas was in the tank?

1. Formula for equation of a line through 2 points:

<u>y-y1</u>	$= \frac{x-x1}{x-x1}$
<i>y</i> 2 – <i>y</i> 1	<i>x</i> 2 – <i>x</i> 1
(90,40)	(360,10)
(x ₁ ,y ₁)	(x ₂ ,y ₂)
$\frac{y-40}{10-40}$	$=\frac{x-90}{360-90}$
$\frac{y-40}{-30}$	$=\frac{x-90}{270}$
$\frac{y - 40}{-1}$ $\frac{y - 40}{-1} = 0$ $\frac{y - 40}{-1} = 0$ $\frac{y - 360}{-1} = 0$ $\frac{y - 360}{-1} = 0$	$= \frac{x - 90}{9}$ = $-1(x - 90)$ -x + 90) 0 + 360

 $9_V = -x + 450$

Expectations

2. This is a linear equation with a negative slope -1/9, meaning the truck burns 1 gallon per 9 miles. It is a non-proportional relationship because it has a y-intercept of 50. Its graph is a descending line.

3. An empty tank means algebraically that y = o. 0 = -1/9 x + 50 1/9 x = 50 $x = 50 \div 1/9$ x = 450

Answer: Upon reaching 450 miles, the tank will be empty.

4. At departure means point x = 0 miles y = -1/9 (0) + 50y = 0 + 50 y = 50

<u>Answer:</u> The equation is $y = -\frac{1}{9}x + 50$

<u>Answer:</u> At departure, the tank has 50 gallons.

Organic-way Mathematics Copyright © 2012



The Library Floor Plan





The Library Floor Plan





Problem 5 Markers and Erasers

On a sale just before the opening of school, a teacher purchased 4 markers and 6 erasers for \$4.90. A few minutes later, she returned to buy from the same stocks 4 markers and 8 erasers for \$6.00. Determine the price of one marker and one erase. Use multiple means.



Use the given box and the can. Justify your response theoretically (via computations) and empirically (via beans or rice)



Problem 7 Coordinated Polygon

1. Review the plotting of the points and their connections forming the quadrilateral ABCD: A(0,3) B(6,9) D(3,0) C(9,6)

2. Prove that ABCD is a parallelogram.

3. Prove that ABCD is a rectangle.



Review: Scaffolding Strategies & Frameworks

- 1. Verbal-Visual-Word Association (VVWA)
- 2. Concept Map
- 2. Sentence Starters / prompts
- 3. Word Etymology & Vocabulary
- 4. KWL Chart
- 5. Anticipatory Guide
- 6. Frayer Model
- 7. Math Glossary Puzzle
- 8. Flow Chart / Table / Diagram
- 9. Math Poetry
- 10. Close Reading
- 11. Socratic Method for Teaching & Learning
- 12. Project-based Learning
- 13. Organic-way Math (under study)
- 14. Math Games (e.g., ORGABEZ)
- 15. Etc.

HOT LINKS

- Common Core Math and ELLs: Blog Posts
- These posts from our Common Core blog highlight resources that can be used in Common Core math instruction for ELLs. EngageNY: Common Core and Math
- The EngageNY website provides materials focused on curricular examples, standards for mathematical practice, and other materials for professional development.
- Discovery Education: Puzzlemaker
- Puzzlemaker is a free puzzle generation tool for teachers, students and parents where users can create and print customized word search, criss-cross, math puzzles, and more using their own word lists.
- Catherine Snow: Word Generation
- Catherine Snow's new website provides information and resources for educators who would like to learn more about Word Generation and how it is implemented. Includes links to comprehensive academic word list that students need to master to comprehend academic content.
- Dave's Math Tables: English/Spanish Dictionary
- List of English math vocabulary words with the Spanish equivalent.
- PBS Teachers: Math Lessons
- PBS Teachers offers a database of multimedia math lessons and activities that can be searched by grade or topic.
- SMART Notebook Lesson Activities
- Browse lesson activities and materials for classrooms using SMART notebook software. Choose the country or region, and search by curriculum standards, subject and grade level.
- "Smart" by Shel Silverstein: Complete poem and activity
- This worksheet offers the complete poem cited in the article, as well as some fun related activities and questions.
- Colorín Colorado Webcast: English Language Learners in Middle and High School
- This webcast features Dr. Deborah Short and discusses effective instructional strategies for teaching English language learner students in middle and high school, such as the SIOP model.
- Translating Word Problems
- This is a great site for teachers in the elementary levels, as it provides a list of keywords you can teach your ELLs to look for as they read word problems. Also included are useful ideas and tricks to better prepare students to understand written math problems.

http://www.colorincolorado.org/article/math-instruction-english-language-learners



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Thank You