



**New York State Statewide Language RBERN at NYU
Citywide Mathematics Professional Development**

726 Broadway, New York, N.Y. 10003, Room 542
Friday, November 16, 2018

**Math Best Practices:
Scaffolding through the Pillars
of School Mathematics
Grades 6 - 8**

by
Archangelo Joseph
Marie-Alix Emmanuel
NYS RBERN Resource Specialists

New York State Statewide Language RBERN at NYU
Citywide Mathematics Professional Development (Grades 6 – 8)

726 Broadway, New York, N.Y. 10003
Friday, November 16, 2018 (9:00 am – 2:00 pm)

Presenters:

Archangelo Joseph

Marie-Alix Emmanuel

Resource Specialists

AGENDA

9:00 - 11:30 am (Morning Session)

Greetings

1. Warm-ups

2. Student Data & PD Rationale

3. Best Practices: Scaffolding through the Pillars of School Mathematics (e.g., Socratic Method of Teaching & Learning, SMTL)
(Inductive Reasoning & Deductive Reasoning)

- Scaffolding through Language (e.g., VVWA, Playlet, and Graphic Organizers)
- Scaffolding through Skills (Procedural Fluency)
- Scaffolding through Conceptual Understanding (Why $\pi = 3.14$?; Why $A = \pi r^2$?; Why $c^2 = a^2 + b^2$?; Why triangle angle sum = 180° ?)
- Scaffolding for Problem Solving (Comparing/Contrasting Capacities; Formulating Equations; Mango Equations & Transformations)

LUNCH 11:30 am – 12:00 pm

12:00 – 2:00 pm (Afternoon Session)

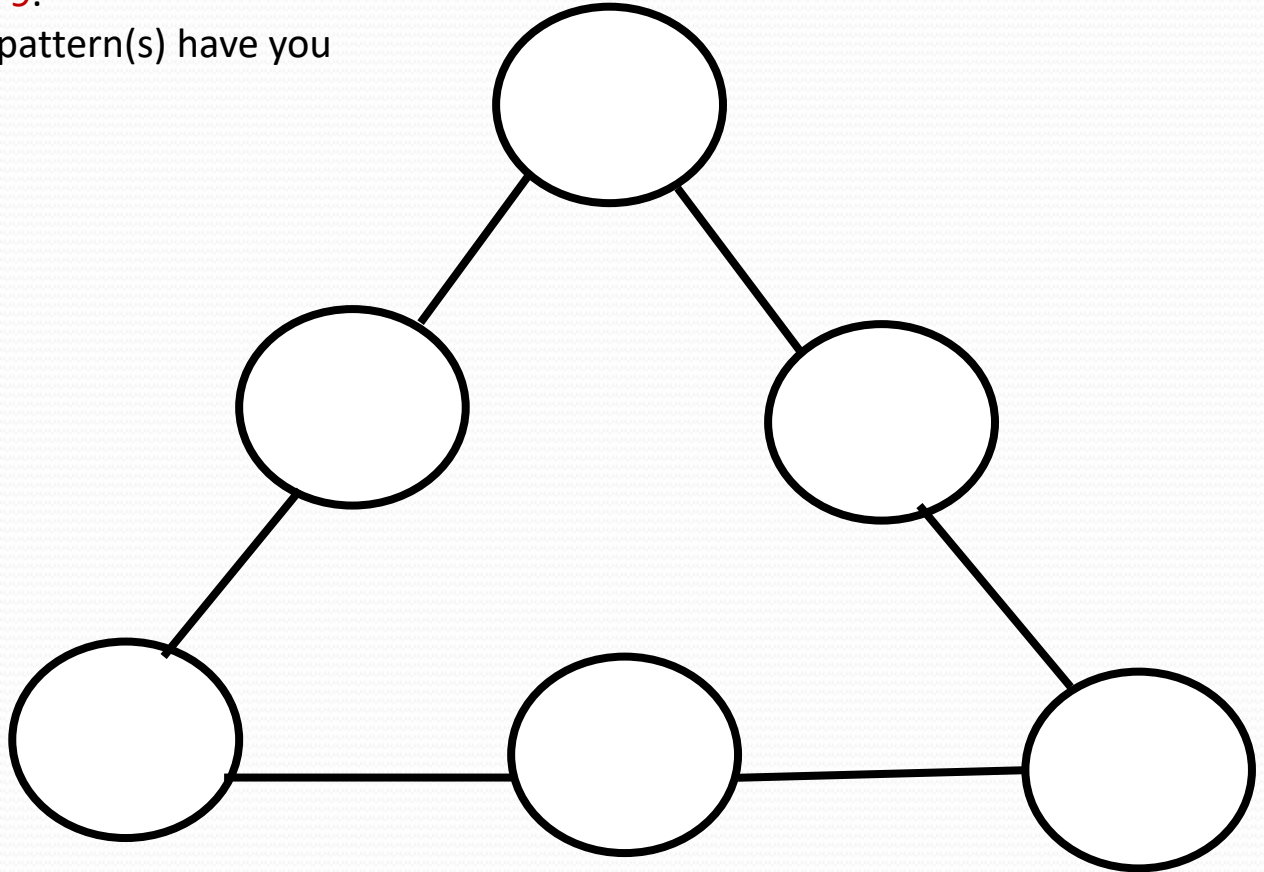
4. Independent Group Work (Application of SMTL)

- Problem Solving, Presentations & Discussions
- Reflections/Implications for Classrooms

5. Evaluation

WARM-UP 1

Place one of these numbers 1, 2, 3, 4, 5, and 6 inside each circle so that the **sum** of the numbers on each side of the “triangle” is **9**. Do not repeat a number. What pattern(s) have you observed?

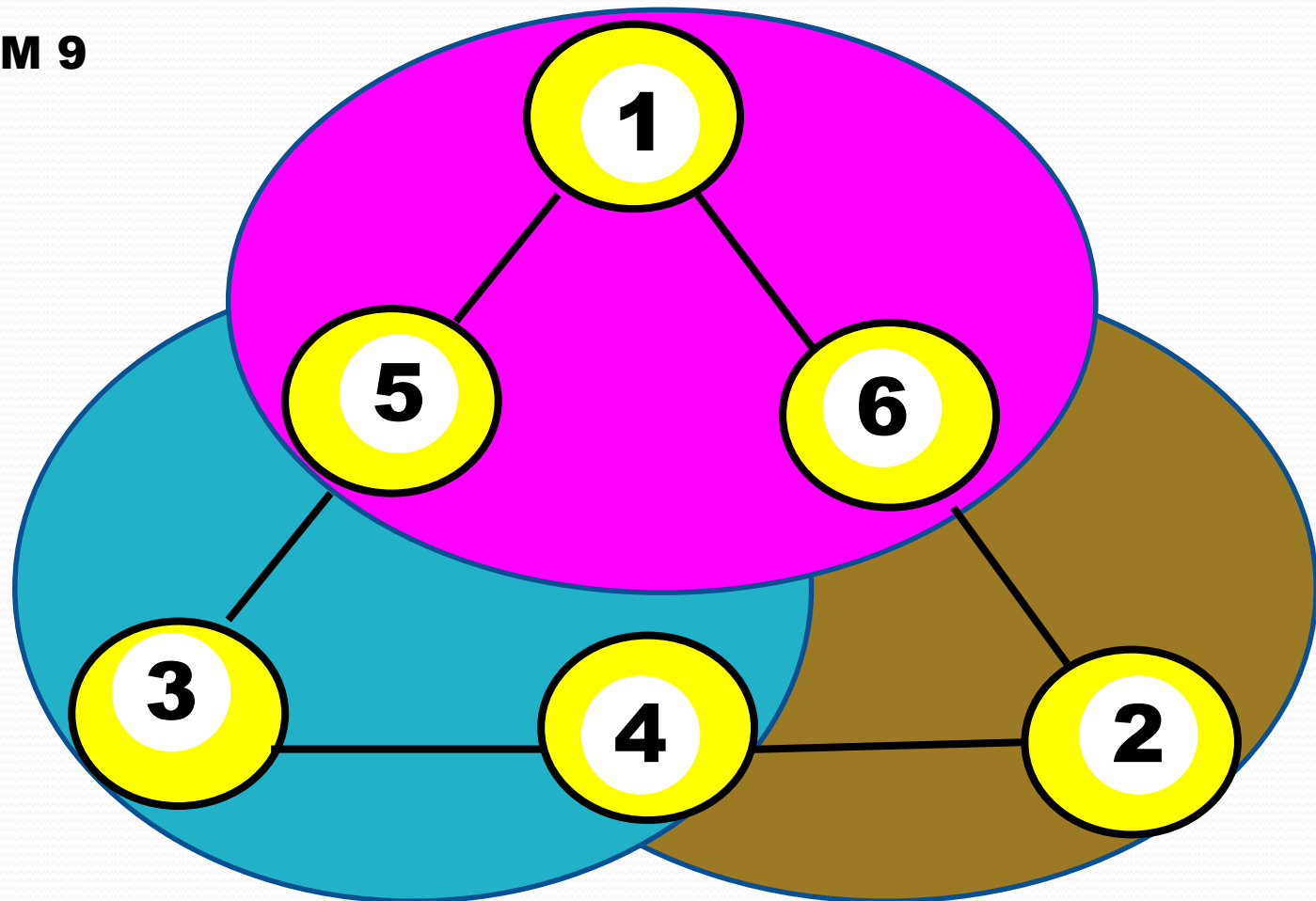


WARM-UP

Place one of these numbers 1, 2, 3, 4, 5, and 6 inside each circle so that the **sum** of the numbers on the side of the “triangle” is **9**.

Do not repeat a number. What pattern(s) have you observed?

SUM 9



Place one of these number in each cell of the grid so that their sum equal 15, horizontally, vertically, or diagonally: 1, 2, 3, 4, 5, 6, 7, 8, 9. Do not repeat a number.

WARM-UP 2

Magic Square

15-SUM

2	7	6
9	5	1
4	3	8

Teacher Self Assessment

Self Assessment Statements	always	often	rarely	never
1. I elicit students' prior knowledge using various means, i.e., realia, visuals, and stories.				
2. I develop and deliver lessons that are rigorous and culturally-relevant to my class.				
3. My classroom is print-rich, picture-rich, respectful, and conducive to learning.				
4. I take time to know my students academically, socially and culturally.				
5. I adopt a class configuration that allows easy access to all my students.				
6. I use probing questions and provide time for responses and cues for scaffolding.				
7. As I plan my lesson., I envision who would need one-on-one assistance.				
8. I take notes of student learning and provide timely and formative feedback.				
9. My lesson plan includes a teacher reflection box that will inform subsequent lessons.				
10. I use best practices including inquiry, constructivism, and the Socratic Learning Method.				
11. I provide ample time for practice, reinforce efforts, and celebrate accomplishments.				
12. I assess the concepts I have taught consistently with the NYS curriculum and standards.				
13. I use the Frayer Model to elicit student prior knowledge and to assess learning (exit slip).				
14. I am on the look out for professional development to further my knowledge on ELLs.				

The 4 Pillars of 21st Century Mathematics

Organic-way Mathematics
(under study-Concordia University)

Concepts

Problems

Skills

Language

Mathematics Discourse Features

Dr. Nancy Cloud

- Are conceptually packed.
- High-density of important words.
- Require up-and-down as well as left-to-right eye movements.
- Require reading-rate adjustment.
- Require multiple readings.
- Use numerous symbolic devices.
- Contain a great deal of technical language with precise meaning.

Decoding the language of math

$+$	$-$	\times	\bullet	\div	$/$	$=$	$?$
\neq	$<$	$>$	\pm	\leq	\geq	\approx	$\%$
\in	\notin	$\#$	\emptyset	\subset	$\not\subset$	\cap	\cup
\therefore	\perp	\Rightarrow	$()$	$[]$	$\{ \}$	Σ	\cong
α	β	Δ	λ	π	Ω	θ	μ

Image 1



www.shutterstock.com · 223184671

Image 2



www.shutterstock.com · 107400536

Image 3



© Can Stock Photo - csp21307088

Image 4



shutterstock

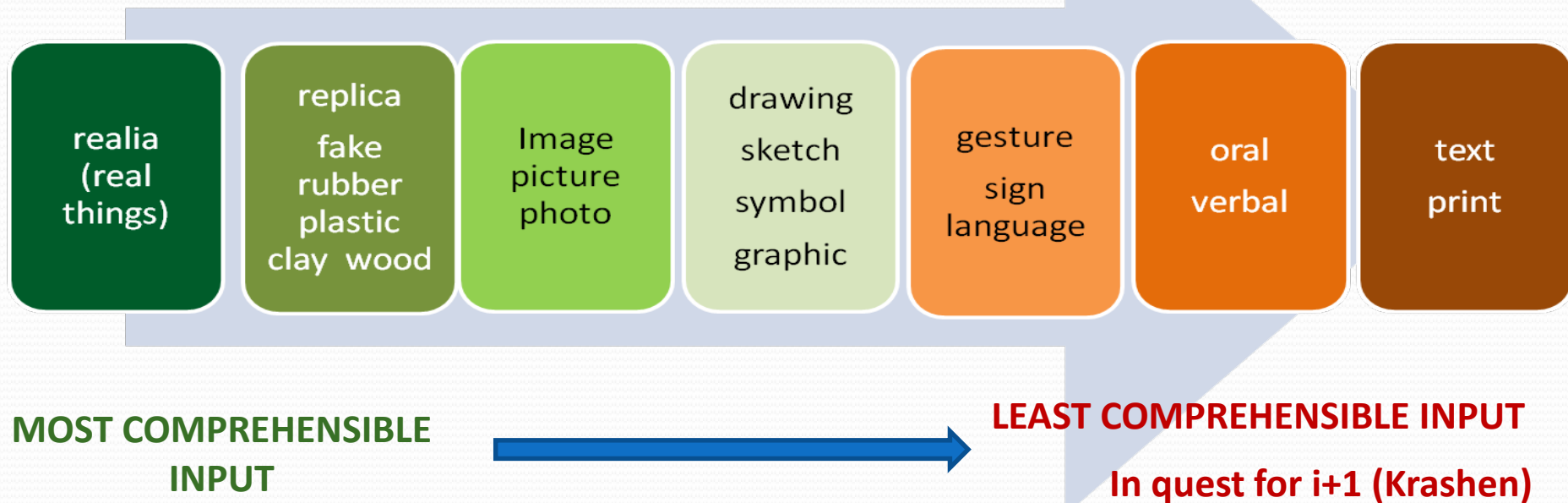
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Focal Concepts

1. Capacity / Volume & Proportion
2. Linear Equation / Function
3. Area & Geometric Transformation

Scaffolding Manipulatives Continuum

From most Concrete to most Abstract





LANGUAGE Scaffolding



Visual-Verbal-Word Association (VVWA)

Visual-Verbal-Word Association

VVWA

Instructions:

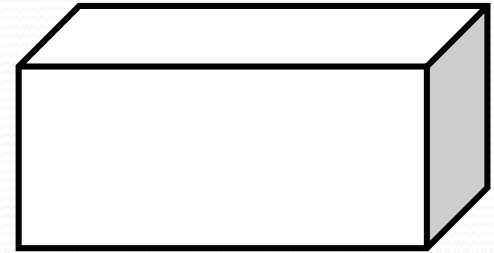
1. Silently identify the symbols/shapes on #1 and # 2.
2. Write both names in your notebook.
3. First complete the declarative sentence on page # 1
4. Then complete the interrogative sentence on # 2.

*** BEGIN**

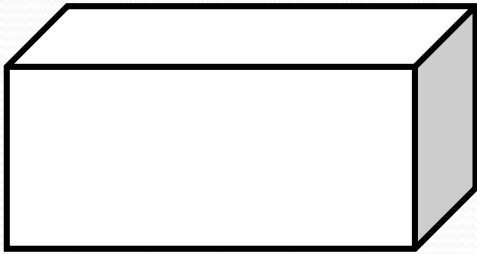
I have ...



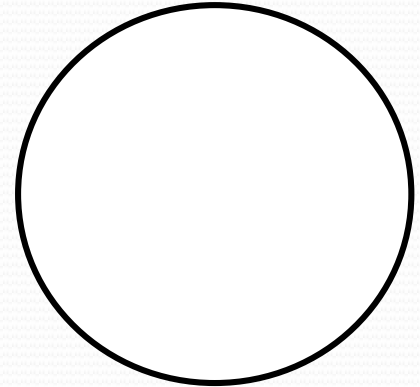
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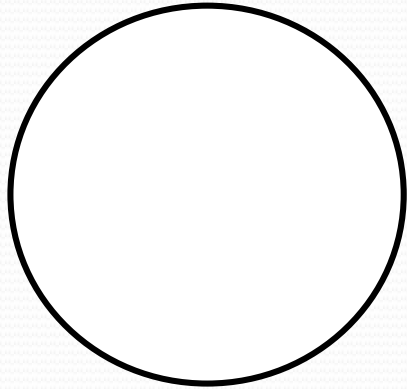
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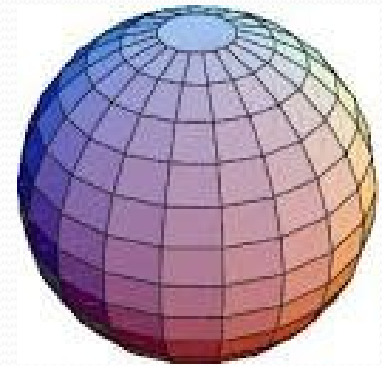
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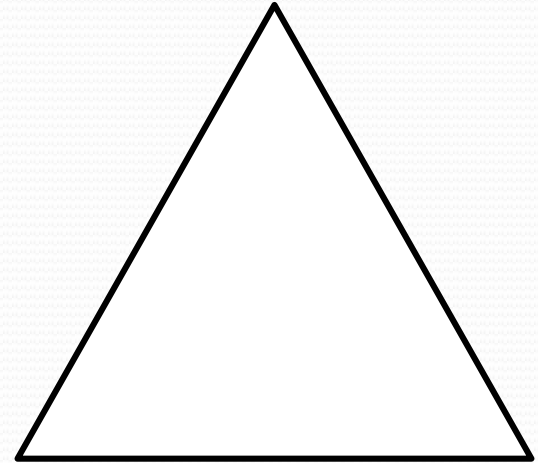
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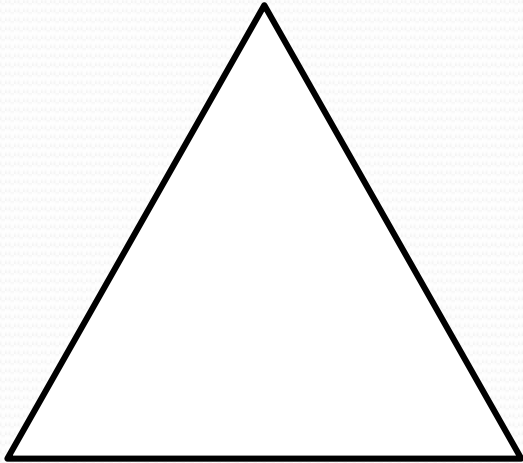
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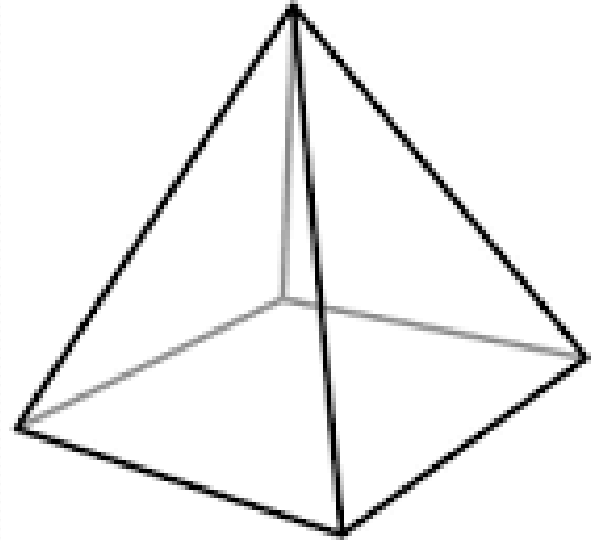
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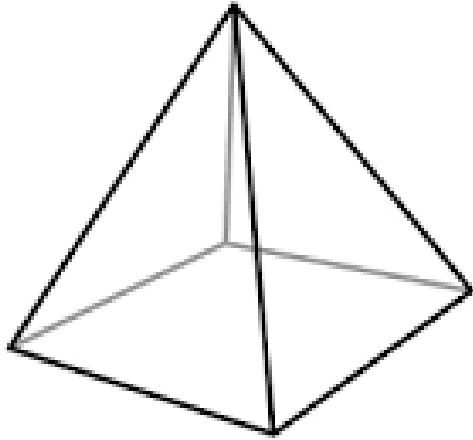
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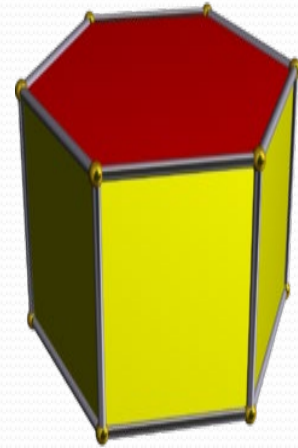
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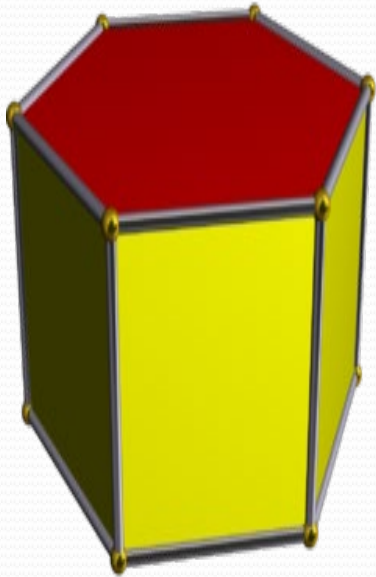
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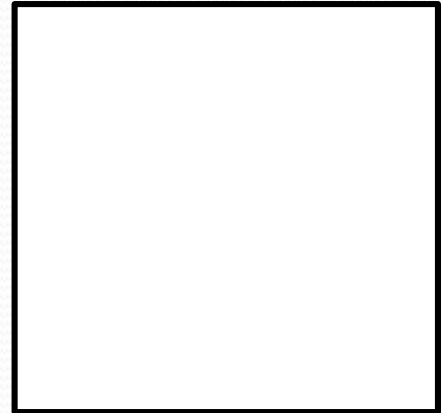
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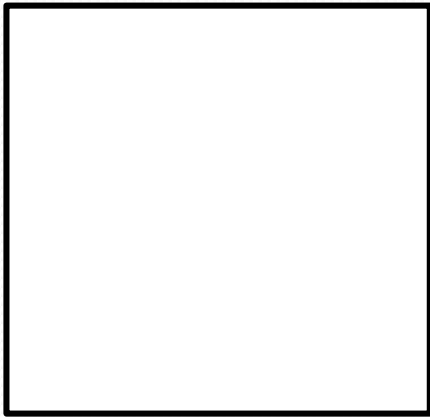
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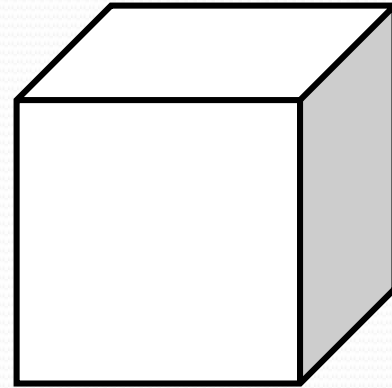
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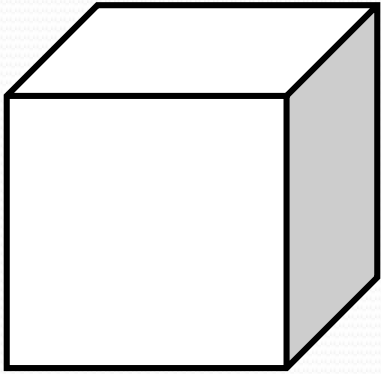
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Who has...?



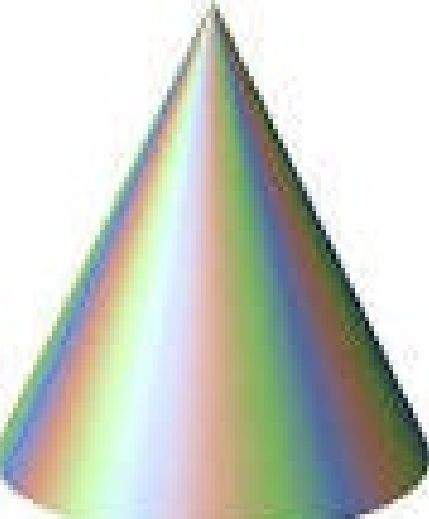
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Who has...?



I have ...



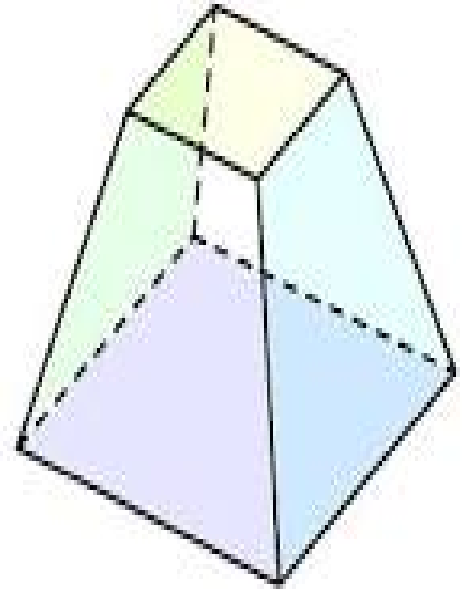
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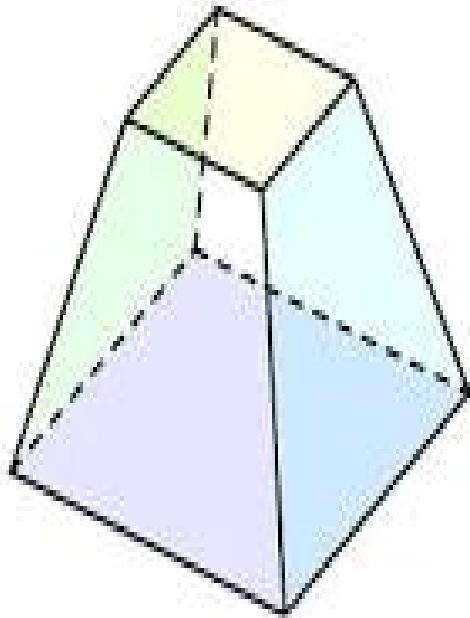
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Who has...?



I have ...



Who has...?

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I have ...

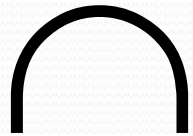
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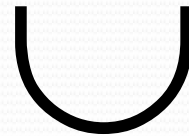
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I have ...



Who has...?

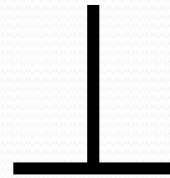




I have ...



Who has...?





I have ...

⊥

Who has...?

//



I have ...

//

Who has...?

$$2x + 3 = 11$$



I have ...

$$2x + 3 = 11$$

Who has...?

$$2x + 3$$

*** END**

I have ...

$$2\chi + 3$$

Sorry!

We are out of
shapes and
symbols

Linguistic modalities covered by the activity How?

Listening	Speaking	Reading	Writing

Anticipatory Guide

Mathematical Statements	Agree	Not sure	Disagree
1. A square has the properties of both rectangle and rhombus.			
2. Any quadrilateral with 4 congruent sides is a square.			
3. The diagonals of a rhombus are perpendicular.			
4. The sum of the measures of all angles in a triangle is 108° .			
5. The quotient of the sum of a and b, and their difference can be written as: $\frac{a+b}{a-b}$			
6. "Three subtracted from five" can be written as " $3 - 5$ ".			
7. The difference between the square of 3 and the double of 3 is 3.			
8. All improper fractions are greater than one.			
9. In the linear equation defined as $y = mx + 3$, x is the slope.			
10. Decimal 0.04785 is greater than decimal 0.4.			
11. The diagonals of a rectangle are always perpendicular.			
12. Constant proportionality means slope (m) equation $y = mx + b$.			

Word Etymology

Volume = volumen (Latin for amount/size of roll/manuscript)

Capacity = capere (Latin for to take; capacitem = breadth, capability of holding much).

Equation = equation (Latin for an equalizing)

Angle = angulus (Latin for sharp bend. Also: ankle)

Polygon = polus (Greek for many) + gōnia (angle, corner)

Acute = acus (Latin for needle, sharp)

Diagonal = dia (Latin for to pass through or join) + gonus (angle)

Diameter = diametros; dia (Greek for pass through or join) + metron (Measure)

Exponent = exos (Latin for out of) + ponere (to place)

Fraction = fractio (Latin for breaking); frangere, to break

Isosceles = iso (Greek for the same) + skelos (legs)

Polyhedron = poli (Greek for many) + hedros (face)

Mono/bi/trinomial = mono = 1 ; bi = 2 ; tri = 3 + nomos = Greek for portion, part

Geometry = geo (Greek for Earth) + metria (measure)

Slope = sleubh (Latin for slip)

Hypotenuse = hypo (Greek for under) + tein (stretch)

Congruent = con (Latin for together) + ruere (fall); congruere (to come together)

Chord = chorde (Greek for string)

Circle = circus (Latin for circular race track)

Kilo = 1,000; **hecto** = 100; **deca** = 10; **deci** = 1/10; **centi** = 1/100; **milli** = 1/1000

Vocabulary

Types of Words

Use a two-column chart to differentiate the one-meaning words from the multiple-meaning words. Explain your thinking.

<i>table</i>	<i>domain</i>	<i>exponent</i>	<i>dividend</i>
<i>volume</i>	<i>root</i>	<i>hypotenuse</i>	<i>gross</i>
<i>power</i>	<i>bank</i>	<i>coefficient</i>	<i>terms</i>
<i>total</i>	<i>odd</i>	<i>equation</i>	<i>trapezoid</i>

Types of Words

One-meaning Words

exponent *total*
equation *coefficient*
trapezoid *hypotenuse*

Multiple-meaning Words

table *domain*
dividend *volume*
root *gross*
power *bank*
terms *odd*

Tiered Words (Word Sophistication)

TIER 1 Basic

more

less

again

share

TIER 2

Academic (across subjects)

additive

subtractive

reproduction

fragmentation

TIER 3

Subject-Specific

addition

subtraction

multiplication

division

Syntax within the Math Register

- The syntax / sentence structure of math can be troublesome. Example: 3 subtracted from 5 equals 2 can be written as $5 - 3 = 2$.

2. At times, key words can be deceiving. Use logic instead. Example:

John has 2 cats and 4 dogs. How many cats does he have in all?

Math Glossary Puzzle

Match the words with their
definitions on the next page. Then
locate them on the grid

TOTAL

ROOT

INTEGER

CHORD

ALL

NO

PROPORTION

REST

SUM

TOP

FACTOR

ORIGINAL

OPT

TOOL

NAME

REPORT

LOCUS

PERCENT

ADD

EPICENTER

PRACTICAL

NONAGON

LAW

What math task (s) can you develop from this letter grid?

P	R	O	P	O	R	T	I	O	N
E	E	R	P	P	O	S	U	M	O
R	S	I	R	T	O	T	A	L	N
C	T	G	A	S	T	O	O	L	A
E	P	I	C	E	N	T	E	R	G
N	I	N	T	E	G	E	R	E	O
T	B	A	I	N	O	T	O	P	N
A	L	L	C	H	O	R	D	O	A
D	A	F	A	C	T	O	R	R	M
D	W	Y	L	O	C	U	S	T	E

Math Glossary Puzzle

Match the words with their definitions on the next page. Then locate them on the grid

TOTAL

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LAW

What math task (s) can you develop from this letter grid?

P	R	O	P	O	R	T	I	O	N
E	E	R	P	P	O	S	U	M	O
R	S	I	R	T	O	T	A	L	N
C	T	G	A	S	T	O	O	L	A
E	P	I	C	E	N	T	E	R	G
N	I	N	T	E	G	E	R	E	O
T	B	A	I	N	O	T	O	P	N
A	L	L	C	H	O	R	D	O	A
D	A	F	A	C	T	O	R	R	M
D	W	Y	L	O	C	U	S	T	E

Math Glossary Puzzle

Word Definitions

- _____. Adjective for beginning
- _____. Synonym for principle or rule
- _____. A number that multiplies another
- _____. A polygon with nine sides
- _____. Antonym for bottom
- _____. Adjective for practice
- _____. Result of addition-Word ending with “M”
- _____. Point on earth at the center of a quake

Math Glossary Puzzle

Word Definitions

- _____. Put things together
- _____. Equality between two ratios
- _____. Meaning “for each 100”
- _____. Antonym for yes
- _____. Meaning “place or position”
- _____. Line segment connecting 2 points on the circumference
- _____. Conclusion from an investigation

Math Glossary Puzzle

Word Definitions

- _____. Appellation (used to designate a person or thing)
- _____. Meaning “everything”
- _____. Base of a power. May also mean part of a plant
- _____. Instrument
- _____. Meaning “choose”
- _____. Synonym for sum
- _____. Any signed number. Example: -5 , $+6$, etc.

Math Glossary Puzzle

Word Definitions

ORIGINAL

. Adjective for beginning

LAW

. Synonym for principle or rule

FACTOR

. A number that multiplies another

NONAGON

. A polygon with nine sides

TOP

. Antonym for bottom

PRACTICAL

. Adjective for practice

SUM

. Result of addition-Word ending with "M"

EPICENTER

. Point on earth at the center of a quake

Math Glossary Puzzle

Word Definitions

ADD

. Put things together

PROPORTION

. Equality between two ratios

PERCENT

. Meaning “for each 100”

NO

. Antonym for yes

LOCUS

. Meaning “place or position”

CHORD

. Segment connecting 2 points on a circumference

REPORT

. Conclusion from an investigation

Math Glossary Puzzle

Definitions

NAME

. Appellation (used to designate a person or thing)

ALL

. Meaning “everything”

ROOT

. Base of a power. May also mean part of a plant

TOOL

. Instrument

OPT

. Meaning “choose”

TOTAL

. Synonym for sum

INTEGER

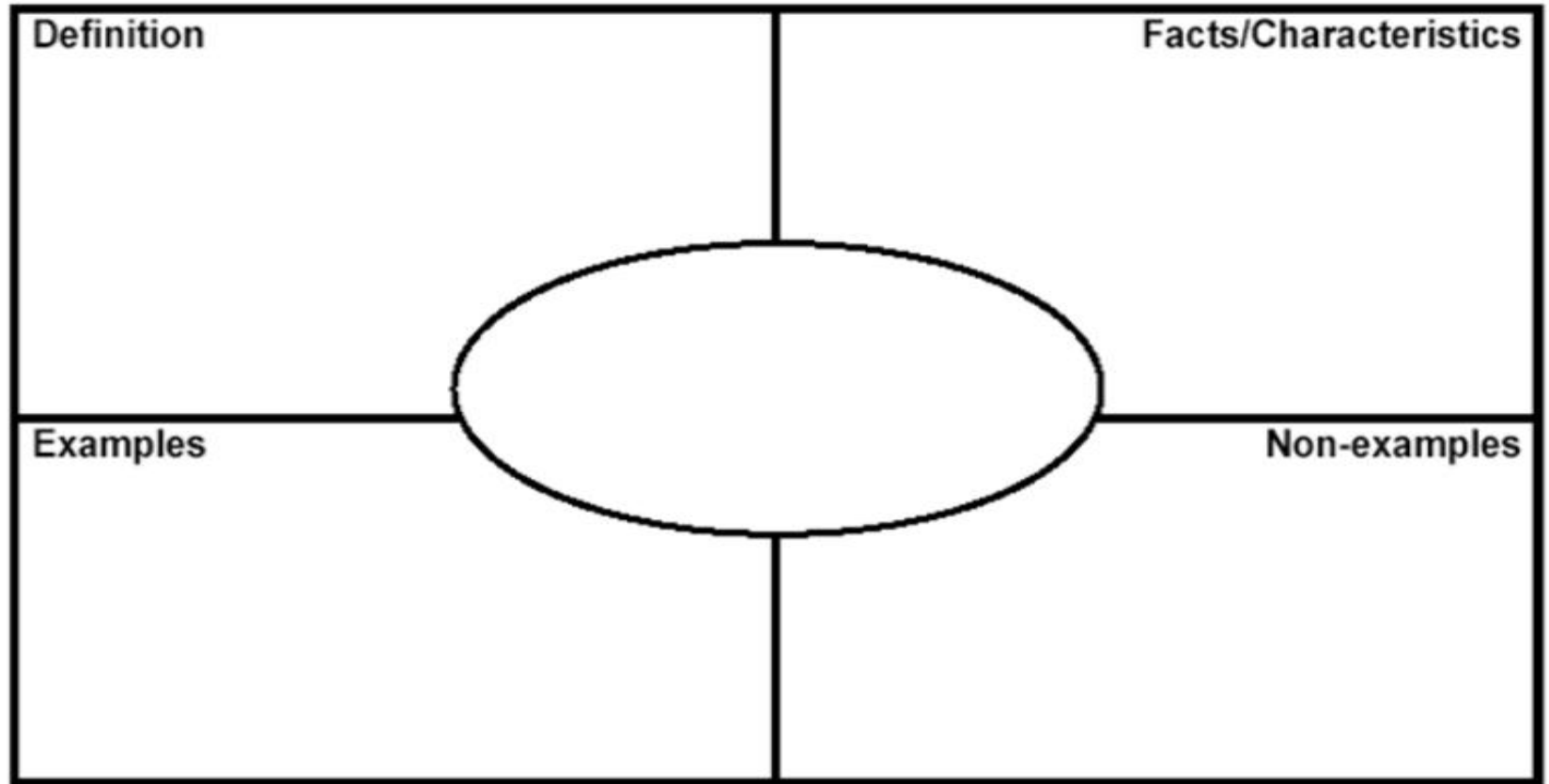
. Any signed number. Example: -5 , $+6$, etc.

Puzzle Website

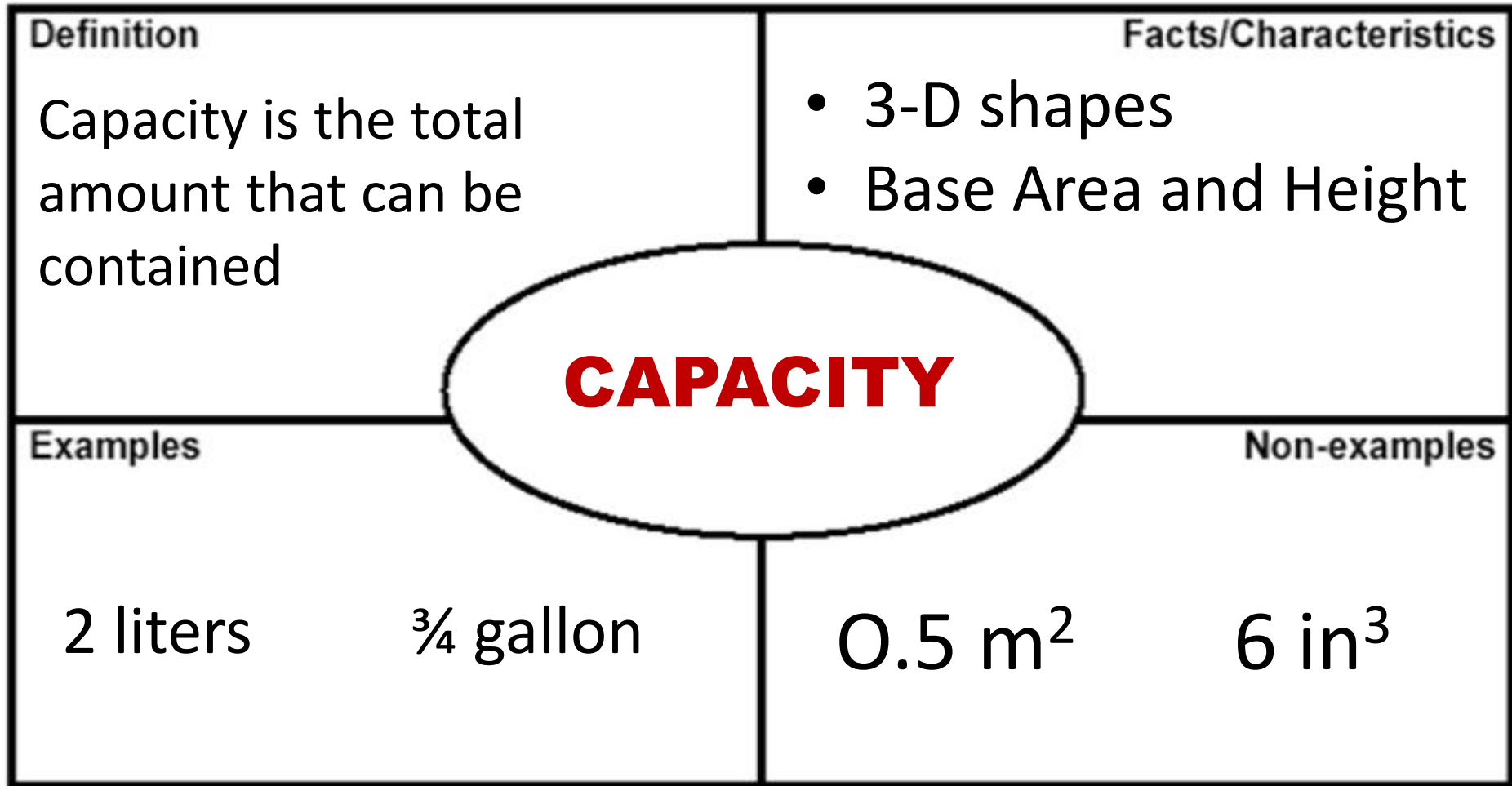
To make your own, go to:

Puzzlemaker.discoveryeducation.com

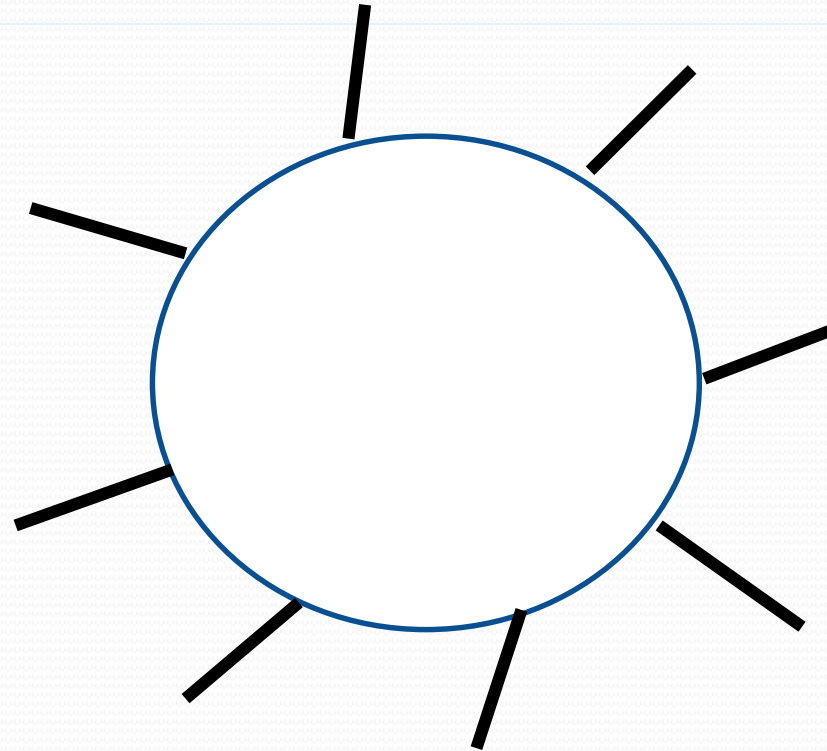
Frayer Model



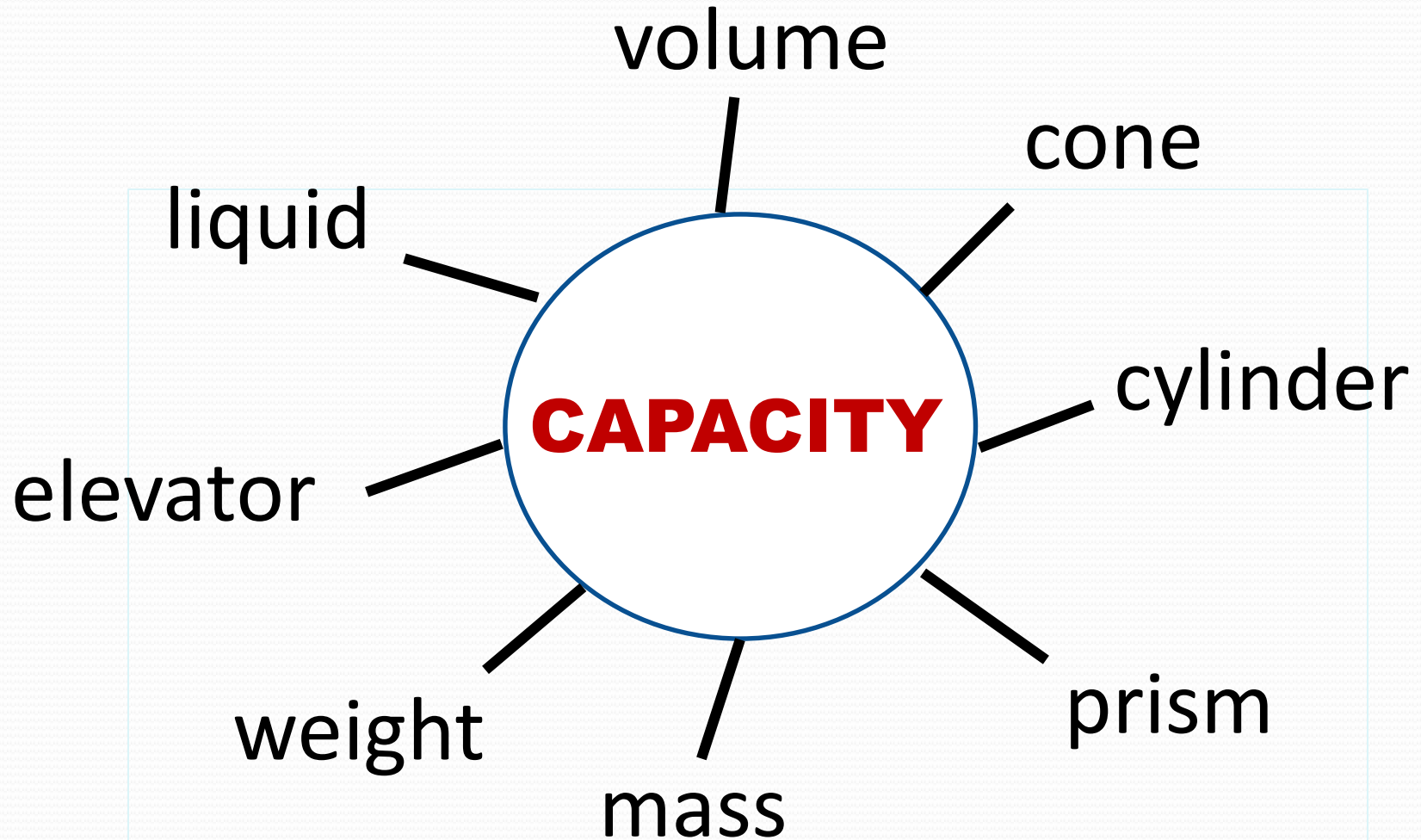
Frayer Model



Concept Map



Simple Concept Map





K

W

L

What I <u>k</u>now PRIOR KNOWLDEGE	What I <u>w</u>ant to know GOAL	What I have <u>l</u>eanerd OUTCOME

K**W****L****What I **k**now****PRIOR-KNOWLEDGE**

I know that capacity is the total amount that can be contained in a 3-D object.

What I **want to know****GOAL
LESSON OBJECTIVE**

How is capacity different from volume?

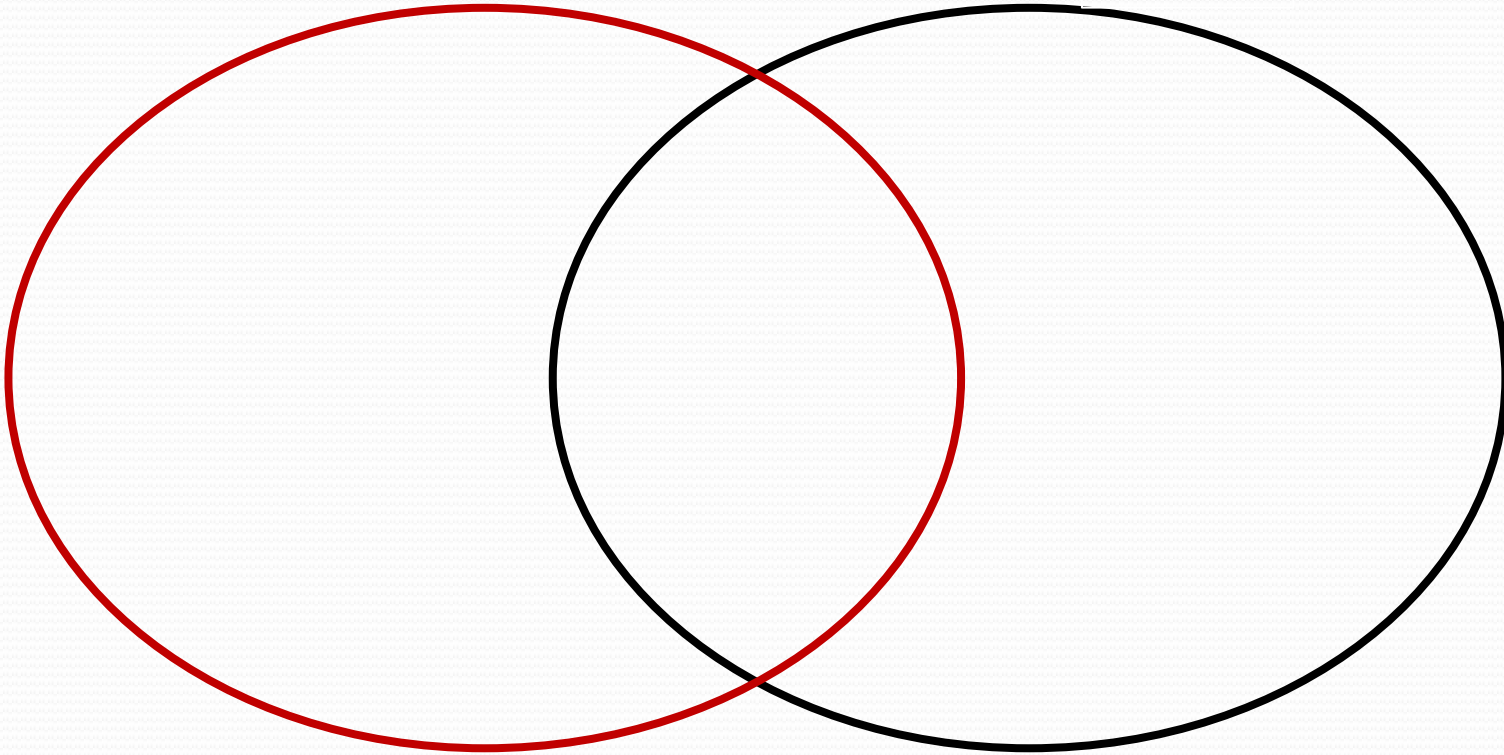
What I have **learned****OUTCOME**

I have learned that capacity is the amount of substance that a 3-D (solid) can contain based on measures from inside, whereas the volume is the space that the solid occupies based on measures from outside.

Venn Diagram

A

B



Venn Diagram

Capacity

Volume

- Space inside of containers
- Liter
- Etc.

- Height
- Base area
- 3-D shapes
- Weight

- Space outside of containers
- dm^3
- Etc.

Anticipatory Guide

Mathematical Statements	Agree	Not sure	Disagree
1. A square has the properties of both rectangle and rhombus.	Y		
2. Any quadrilateral with 4 congruent sides is a square.			N
3. The diagonals of a rhombus are perpendicular.	Y		
4. The sum of the measures of all angles in a triangle is 108° .			N
5. The quotient of the sum of a and b, and their difference can be written as: $\frac{a+b}{a-b}$	Y		
6. "Three subtracted from five" can be written as " $3 - 5$ ".			N
7. The difference between the square of 3 and the double of 3 is 3.	Y		
8. All improper fractions are greater than one.	Y		
9. In the linear equation defined as $y = mx + 3$, x is the slope.			N
10. Decimal 0.04785 is greater than decimal 0.4.			N
11. The diagonals of a rectangle are always perpendicular.			N
12. Constant proportionality means slope (m) equation $y = mx + b$.	Y		

Math Riddles

1. I am a polyhedron 8 vertices, 12 edges, and 6 faces. Who am I?
2. I am a space figure with no vertex like the globe than fifteen. Who am I?
3. I am a number that is ten less than fifty. Who am I? Write my equation.
4. I am an even number. If you add six to me, and then subtract two, the result will be eighteen. Who am I?
5. I belong to a family of numbers having only two factors. Who am I?
6. In a fraction, I am the number on top of the bar. Who am I?
7. I am the borderline of enclosed shapes. Sometimes, students mistake me for area. Who am I.?
8. I am the identity property for multiplication and division. Who am I?
9. I am the identity property for addition and subtraction. Who am I?
10. I am the inverse operation to addition. Who am I?
11. I am the inverse operation to division. Who am I?
12. I am simultaneously the square of 3 and one-half of 18 . Who am I?

Math Riddles

1. I am a quadrilateral with two pairs of parallel sides. Who am I? **RECTANGULAR PRISM**
2. I am a space figure with no vertex like the globe than fifteen. Who am I? **SPHERE**
3. I am a number that is ten less than fifty. Who am I? Write my equation. **40** **$n = 50 - 10$**
4. I am an even number. If you add six to me, and then subtract two, the result will be eighteen. Who am I? **14**
5. I belong to a family of numbers having only two factors. Who am I? **PRIME NUMBER**
6. In a fraction, I am the number on top of the bar. Who am I? **NUMERATOR**
7. I am the borderline of enclosed shapes. Sometimes, students mistake me for area. Who am I.? **PERIMETER**
8. I am the identity property for multiplication and division. Who am I? **1**
9. I am the identity property for addition and subtraction. Who am I? **0**
10. I am the inverse operation to addition. Who am I? **SUBTRACTION**
11. I am the inverse operation to division. Who am I? **MULTIPLICATION**
12. I am simultaneously the square of 3 and one-half of 18 . Who am I? **9**

Story Boarding

Examine the next slide.

Write a relevant caption underneath each set of images, and formulate a short story problem.

Story Boarding

1



2



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3



6



5



4



Playlet



Pam's Apple Sauce



Narrator: Welcome Pam and Antonio. Pam is a customer who wants some apples to make applesauce. Antonio is a store keeper. Today, Pam walks into Antonio's Fruit Store, and starts the conversation.

Pam: Good morning, Antonio!

Antonio: Good morning, Pam! How can I help you today?

Pam: I need some apples to make applesauce.

Antonio: That's a good idea. Today, they are fresh and delicious.

Pam: How do you sell the red apples?

Antonio: Two for three.

Pam: All right! I take five

Narrator: Freeze! How much does Pam owe for the red

Antonio: Umm! Let's see... \$_____.

Pam: Ok! I need green apples, too.

Antonio: That's a good idea. Today, they're sweet and

Pam: Is it the same price?

Antonio: Nope!

Pam: How do you sell them?

Antonio: Three for two.

Pam: All right! I take twelve.

Narrator: Freeze! How much does Pam owe for the green apples?

Antonio: Umm! Let's see... \$_____.

Pam: Ok. That's all.

Pam: Here is a twenty-dollar bill. Keep the change!

Narrator: Freeze! How much change (if any) has Pam left behind? Explain.

Antonio: Umm! Let's see..._____. Thank you.



Playlet Mango Juice



***Narrator:** Ladies and gentlemen, please welcome Jeanne and Mary. Jeanne is a young lady who loves mango juice. Mary is a peddler who wants to sell as many mangoes as possible. This morning, out goes Jeanne in search of her favorite fruit. As she crosses a street corner, a soft voice breaks the silence in her mind.*

Mary: My dear beautiful lady, would you come and check my mangoes out?

Jeanne: Ok. No problem. Indeed, I do need some mangoes to make some juice.

Mary: That's a good idea. Today they're fresh and delicious.

Jeanne : How do you sell these *Francique* mangoes?

Mary: Five for three.

Jeanne : Ok. No problem. I'm taking three dozen.

***Narrator:** Freeze! How much does Jeanne owe for the Francis mangoes?*

Mary : Mmm! Let's see... \$_____.

Jeanne : Ok! I also need some *Cinnamon* mangoes.

Mary: That's a good idea. Today they're sweet and juicy.

Jeanne : Is it the same price?

Mary: Nope! These are special. Five for four.

Jeanne : Ok. No problem. I'm taking three and a half dozen.

***Narrator:** Freeze! How much does Jeanne owe for these Cinnamon mangoes?*

Mary : Mmm! Let's see... \$_____

***Narrator:** Freeze! How much does Jeanne owe in all?*

Mary : Mmm! Let's see... \$_____

Jeanne : Ok. No problem. Here's a crispy fifty-dollar bill. Keep the change!

***Narrator:** Freeze! How much change (if any) has Jeanne left for Mary? Explain.*

Mary : Mmm! Let's see... \$_____



©mycolombianrecipes.com

Mango Juice Sample Questions

1. Who is Mary? Why does she want the mangoes for?
2. Who is Jeanne? What's her desire?
3. What is a peddler?
4. Who begins the conversation? Jeanne or Mary? Cite some textual evidence.
5. Which mango brand is more expensive? By how much?
6. What adjectives are used to determine the Francique mangoes?
7. What adjectives are used to describe the Cinnamon mangoes?
8. Why does Jeanne use the determinant "crispy" to characterize her 5-dollar bill?
9. How much change has Jeanne left for Mary? How do you know?
10. What is your impression of Mary? Jeanne? The entire Playlet?
Rewrite the story in your own words, in prose.

Revelation of a Polygon

Blessed am I with some special qualities!
Mathematicians would say unique properties.
'Cause my two **diagonals** are **perpendicular**,
Thus definitely anointing me in particular.

Above all, I am a **full-fledged** Polygon.
From my **vertices**, multiple sides can be drawn.
Moreover, I do inherit traits of Parallelogram,
'Cause parallelism sprawls all over my diagram.

Any clone of mine **inevitably** is a Rectangle,
'Cause each angle is 90 degrees, a **right angle**.
But don't be fooled! All rectangle copies
Don't necessarily possess my qualities.

To some extent, I do look like a **Rhomboid**.
Also, I share a thing or two with a **Trapezoid**.
Definitely, I am a **Rhombus**; my sides are the same.
Oh, boy! Can't you still guess my name.

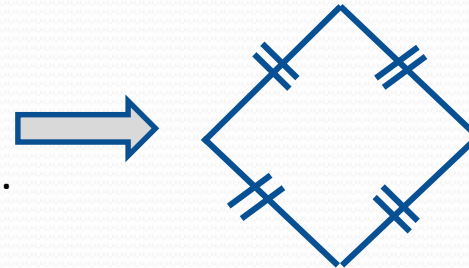
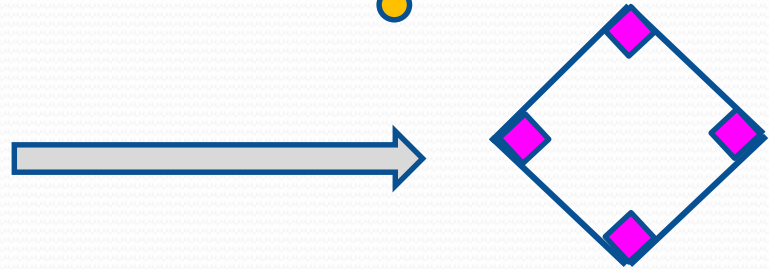
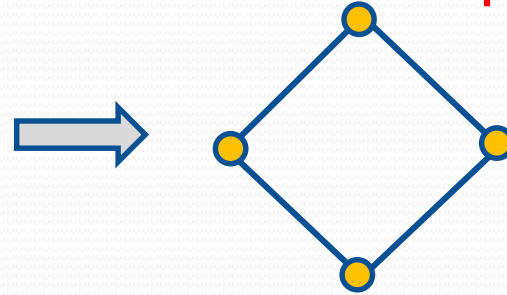
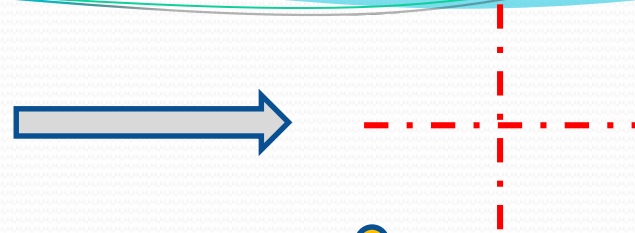
Revelation of a Polygon

Blessed am I with some special qualities!
Mathematicians would say unique properties.
'Cause my two **diagonals** are **perpendicular**,
Thus definitely anointing me in particular.

Above all, I am a **full-fledged** Polygon.
From my **vertices**, multiple sides can be drawn.
Moreover, I do inherit traits of Parallelogram,
"Cause parallelism sprawls all over my diagram.

Any clone of mine **inevitably** is a Rectangle,
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Definitely, I am a **Rhombus**; my sides are the same.
Oh, boy! Can't you still guess my name.



Sample ELA/ENL Questions

1. Which specific polygon is the poem talking about? How do you know? Cite textual evidence.
2. Underline the bolded words and explain their meanings
3. What does “to some extent” mean?
4. Underline all adverbs in the poem and write down their meanings.
5. Rewrite each stanza in your own words.
6. Formulate your own questions.

Close Reading

Anatomy of Problem Solving


Bottle Redemption

Two days following her birthday party, Marie **returned** into the neighborhood store where she had **purchased** the **beverages**. She wanted to **redeem** the bottles. Once there, she **dashed** to the machine and began **inserting** the **empties**. When she finished, she pressed a button, and a total of one **dozen** nickels and dimes **chimed** into a shiny metal tray at the bottom. The **monetary mixture** **amounted** to one dollar. Astonishingly, Marie **realized** that there were **twice as many** dimes as nickels.

What activities would you customize for your students and in what subject areas?

Sample Questions

- Who went to the neighborhood store?
- When did Marie go to the store?
- What did Marie do after her birthday?
- Where did Marie insert the empty bottles?
- How did Marie go to the store?
- Why did Marie return to the store?
- Name one synonym for “beverages.”
- What does the word “redeem” mean?
- What is the value of one dime?

- 
- What does the word “mixture” signify?
 - How do you interpret “twice as many?”
 - What is the value of one nickel?
 - What is “one dozen?”
 - Why does the author use the perfect past tense “had purchased” instead of the simple past tense “purchased?”

Learning Log

- Journal entries (on what has been learned) specified by the teachers
- Not all journal entries are learning logs.



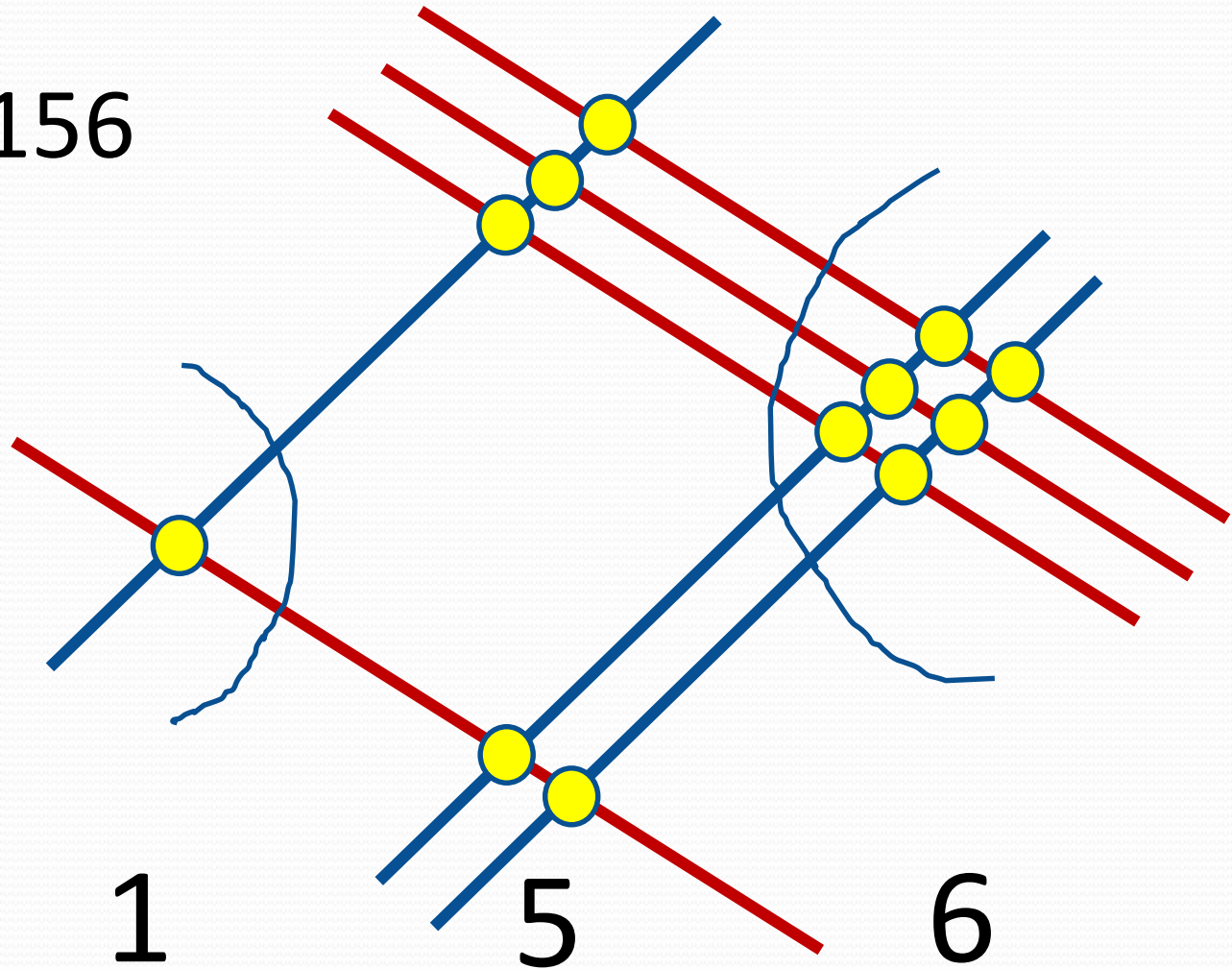
SKILLS

Scaffoldings

Computational Fluency = Speed + Accuracy

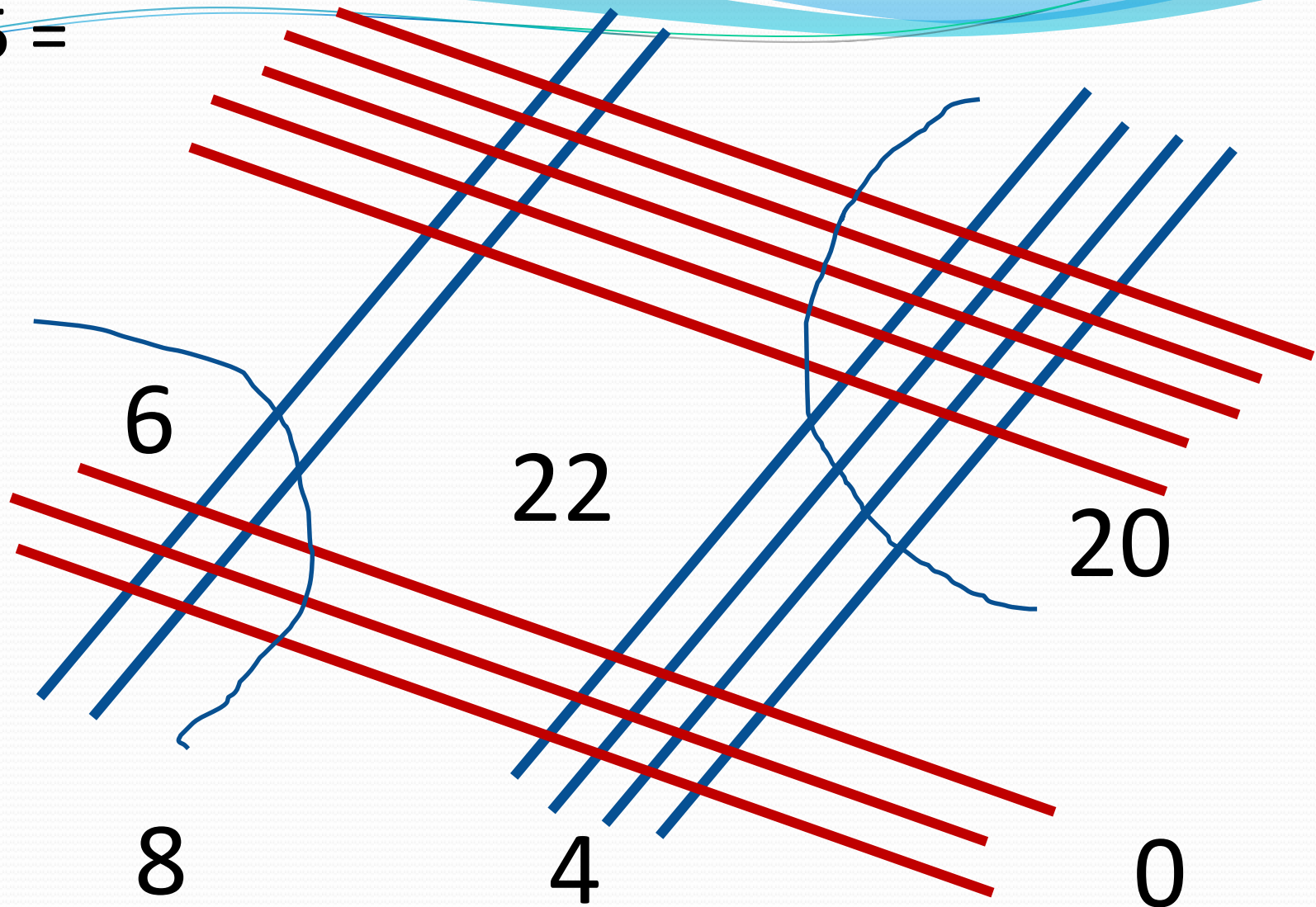
Asian Multiplication

$$12 \times 13 = 156$$

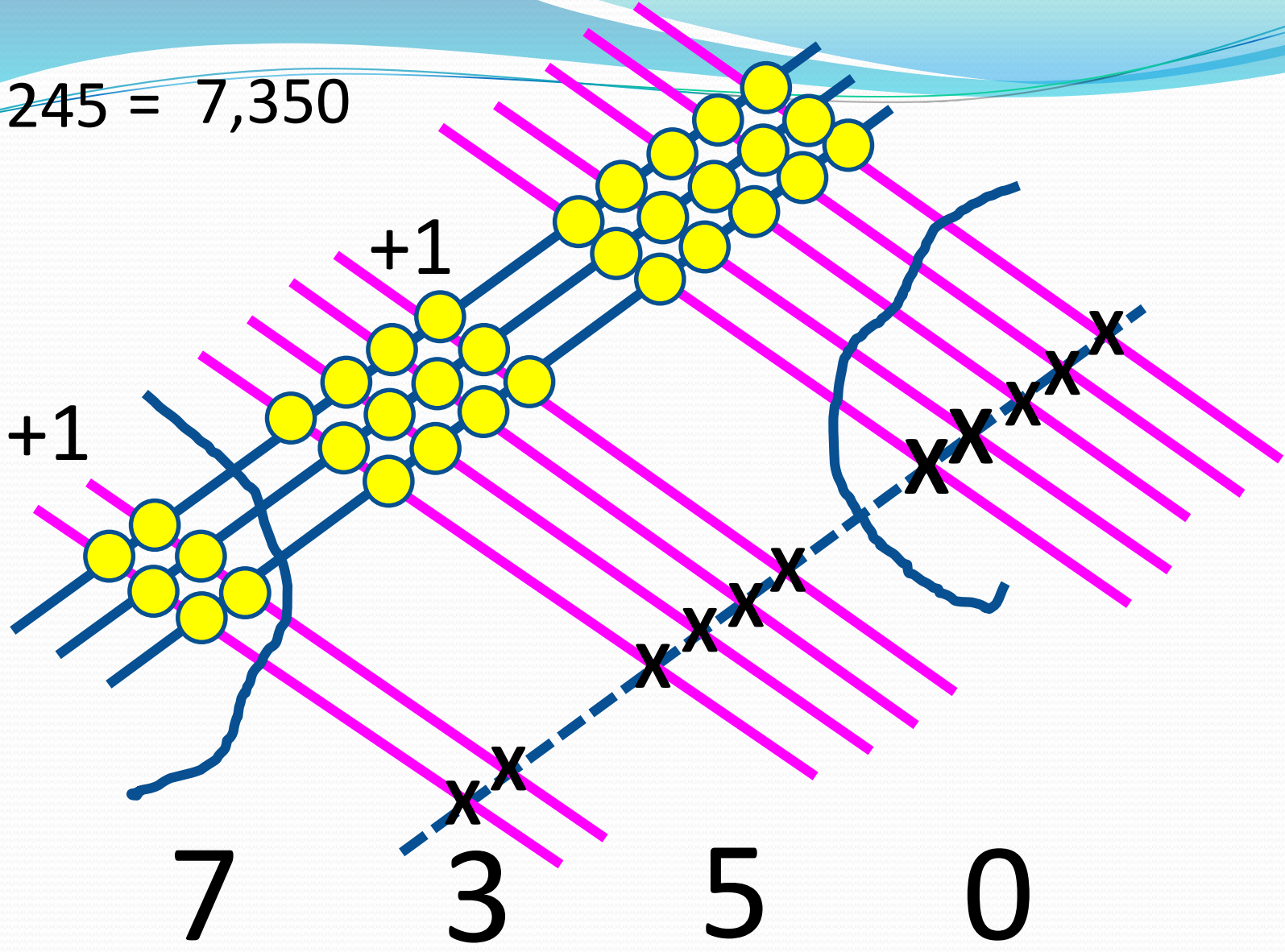


$$24 \times 35 =$$

840



$30 \times 245 = 7,350$



Haitian Division

$$480 \div 15 = 32$$

dividend	divisor
4 8'0	1 5
0 3 0	3 2
0 0	quotient
remainder	

Metric System Prefixes

kilo = 1,000

hecto = 100

deca = 10

Standard = 1 (meter, gram, liter)

deci = .1 = $\frac{1}{10}$

centi = .01 = $\frac{1}{100}$

milli = .001 = $\frac{1}{1000}$

System Metric Table for Length

km	hm	dam	m	dm	cm	mm
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
1000	100	10	1	.1	.01	.001
7	4	6	5			

$$7465 \text{ m} = \underline{74.65} \text{ hm}$$

System Metric Table for Area

km ²	hm ²	dam ²	m ²	dm ²	cm ²	mm ²
05	00	00	00			

$$5 \text{ km}^2 = \underline{5,000,000} \text{ m}^2$$


Note: $1 \text{ km}^2 = 1,000,000 \text{ m}^2$

System Metric Table for Volume/Capacity

km ³	hm ³	dam ³	m ³	dm ³	cm ³	mm ³
005	000	000	000			

$$5 \text{ km}^3 = \underline{5,000,000,000} \text{ m}^3$$

Note: $1 \text{ km}^3 = 1,000,000,000 \text{ m}^3$



Scaffolding for Conceptual Understanding

The Socratic Method

Productive talk is:

- **Deep:** The specific idea being discussed is central to the lesson, presented in interconnected ways, and engages students' analytical thinking.
- **Sustained:** one student's statement is followed by another student's response, which extends, refutes, or questions what was first said.
- **Student-controlled:** Students (not teachers) control what they say. But teachers set the parameters for interactions, sometimes framing questions that start the conversation. These questions are intended to communicate new related ideas, propose counter ideas or counter-examples, and generally enhance the discussion.

The Socratic Method of Teaching & Learning

1. Hypothesis / Claim / Statement
2. Clarification of the Hypothesis
3. Experimentation / Proof
4. Validation / Adjustment /
Rejection of hypothesis

Clarifying Questions

- Factual questions of fact clarifying the dilemma, and providing the nuts and bolts to probing questions

Examples

1. Is this what you said...?
2. Did I hear you say...?
3. Did I understand you when you said...?
4. What criteria did you use to...?
5. What's another way you might...?
6. Did I paraphrase what you said correctly?

Adapted from the Iowa Peace Institute Message

Probing Questions

Inquisitive questions intended to help the presenter think more deeply about the issue at hand.

Examples

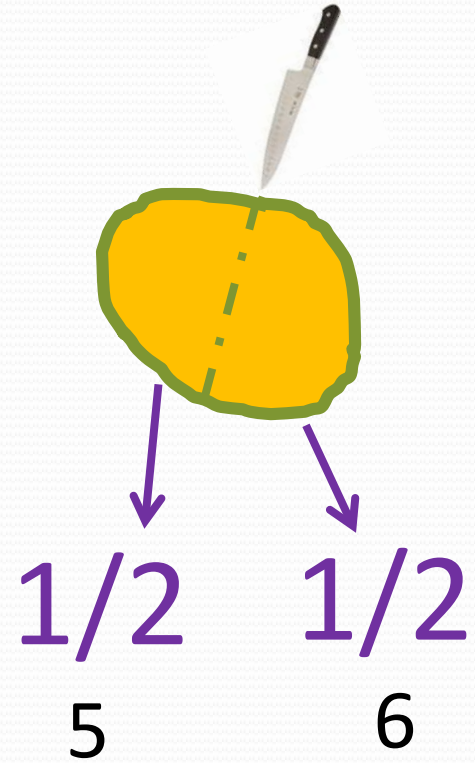
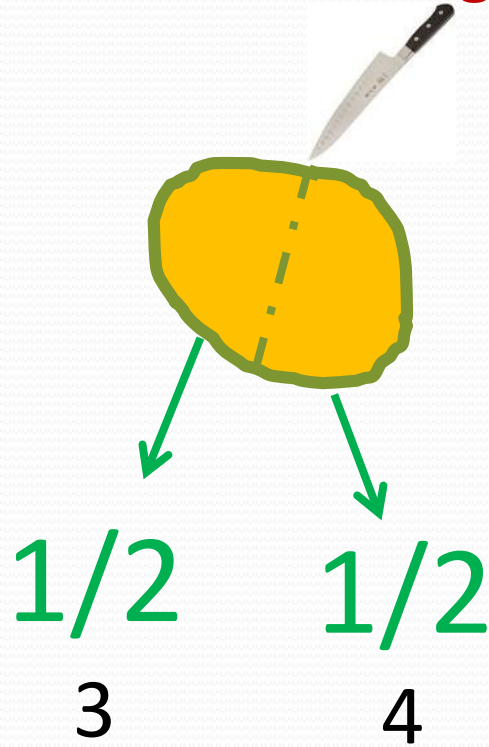
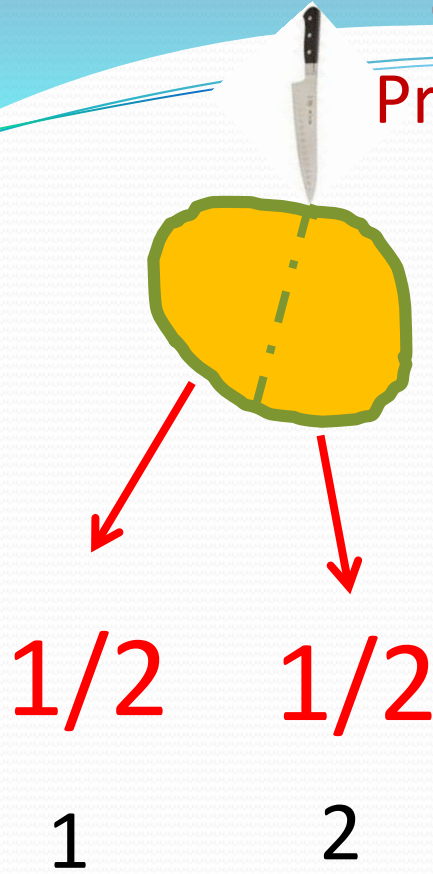
1. Why do you think this is the case?
2. What do you think would happen if...?
3. What sort of impact do you think...?
4. How did you decide...?
5. How did you conclude...?
6. How did you determine...?
7. What is the connection between... and...?
8. What if the opposite were true? Then what?

The Meaning of Division of Fractions

Why $3 \div \frac{1}{2} = 6$?

- What does it mean to divide 3 by $\frac{1}{2}$?
- What real-life situation would require the division above?

Concrete Division for Profound Understanding



Answer: 6 

Six halves $\frac{1}{2}$. Not mangoes.

Why multiply by the reciprocal
when dividing fractions?



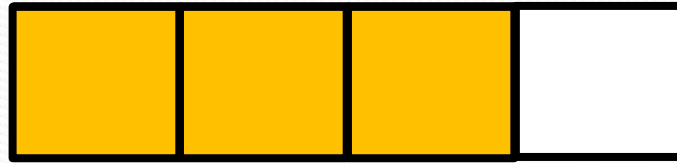
$$3 \div \frac{1}{2} =$$

$$\frac{\frac{3}{1}}{\frac{1}{2}} = \frac{\frac{3}{1} \times \frac{2}{1}}{\frac{1}{2} \times \frac{2}{1}} = \frac{\frac{3}{1} \times \frac{2}{1}}{\cancel{1}} = \frac{3}{1} \times \frac{2}{1} =$$

6

Singapore Model

$$\frac{3}{4} \div \frac{1}{8} = 6$$



$\frac{3}{4}$

\div



$\frac{1}{8}$



Answer: There are 6 eighths in three-fourths.

Why $10^0 = 1$?

- Multiplication of powers: $10^3 \times 10^2 = 10^{(3+2)} = 10^5 = 10,000$

Division of powers: $10^3 \div 10^2 = 10^{(3-2)}$

$$10^3 \div 10^2 = 10^1 = 10$$

The same principle applies to:

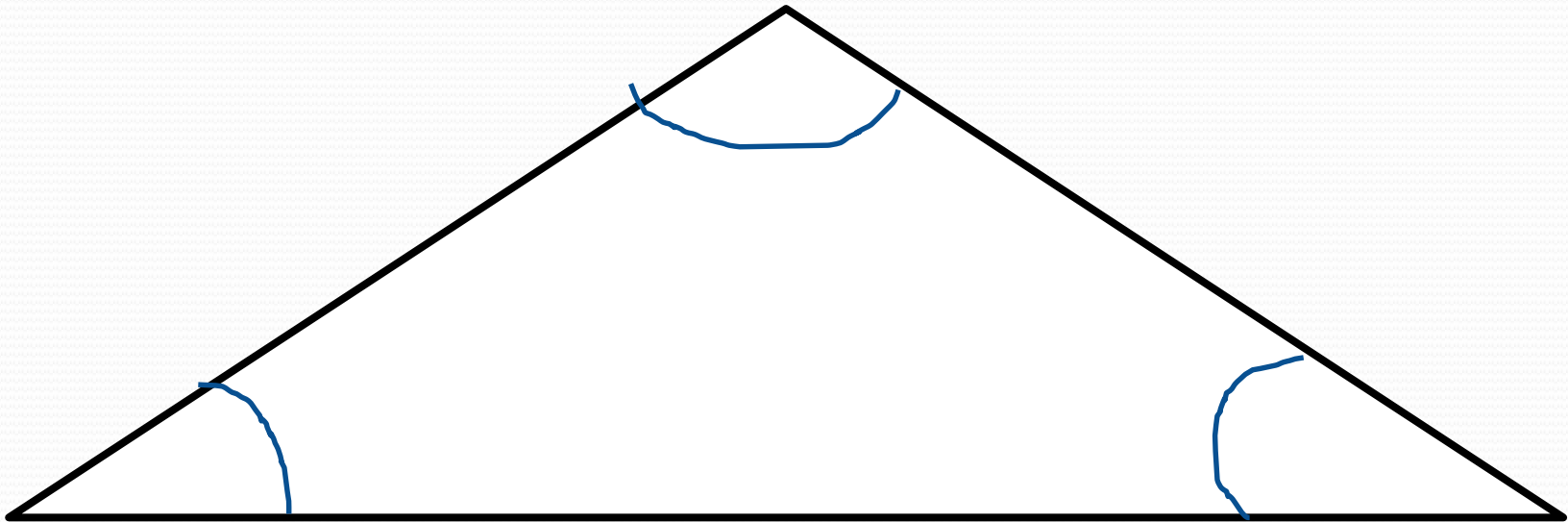
$$10^3 \div 10^3 = 10^{(3-3)}$$

$$10^3 \div 10^3 = 10^0$$

$$10^3 \div 10^3 = 1$$

So, $10^0 = 1$

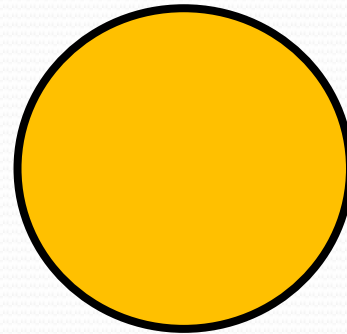
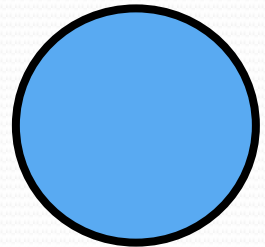
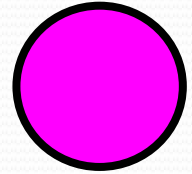
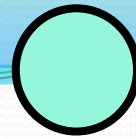
Why the sum of the measures of all angles inside a triangle equals 180° ?



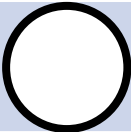
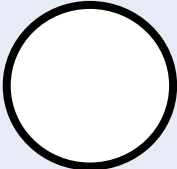
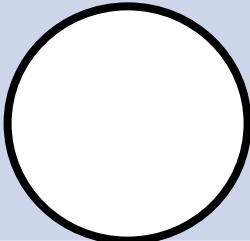
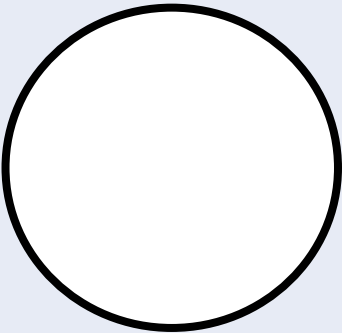
Why $\pi = 3.14$?

The Meaning of Pi.

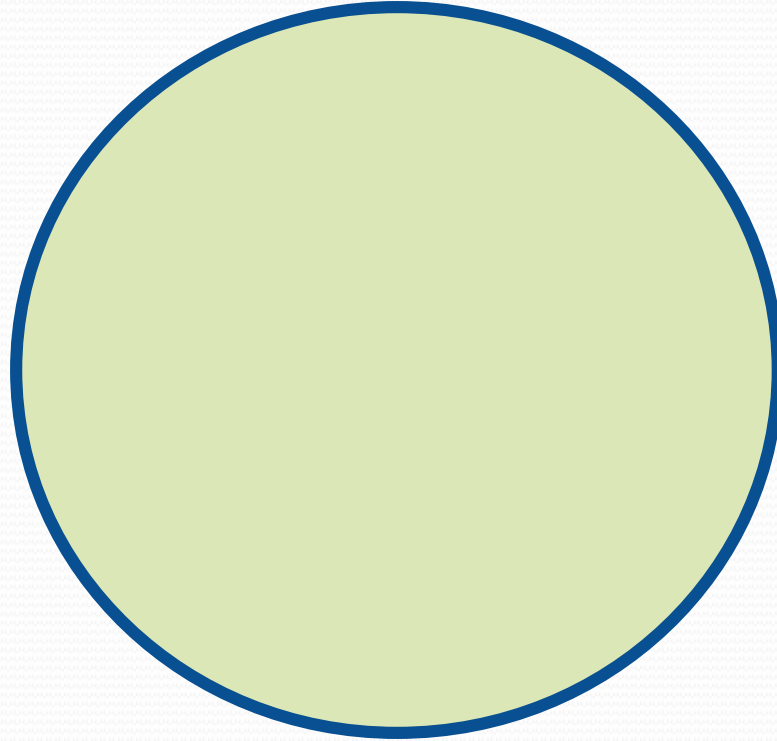
Use tape measurement to determine the circumference and the diameter of your given circle. Then submit your findings to the class to complete the collaborative table (next slide).

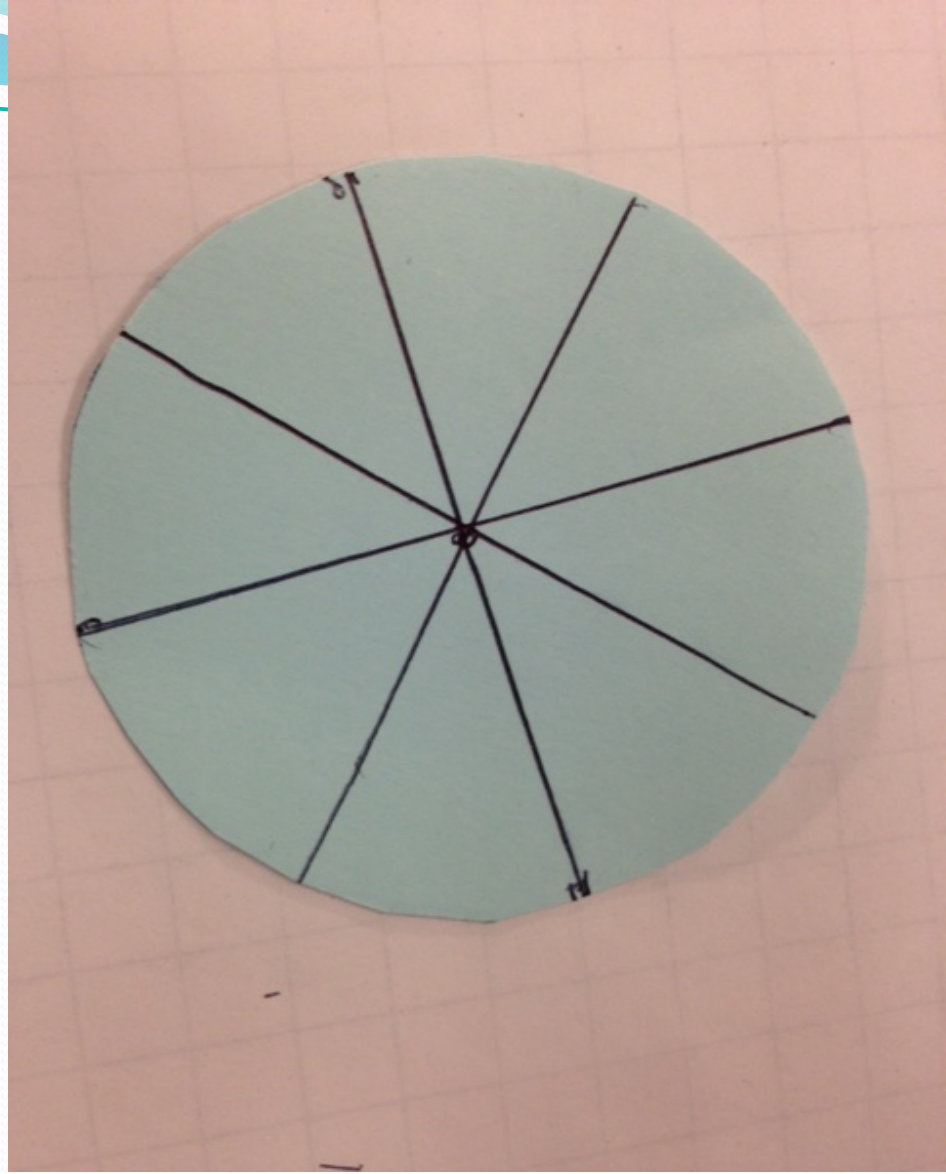
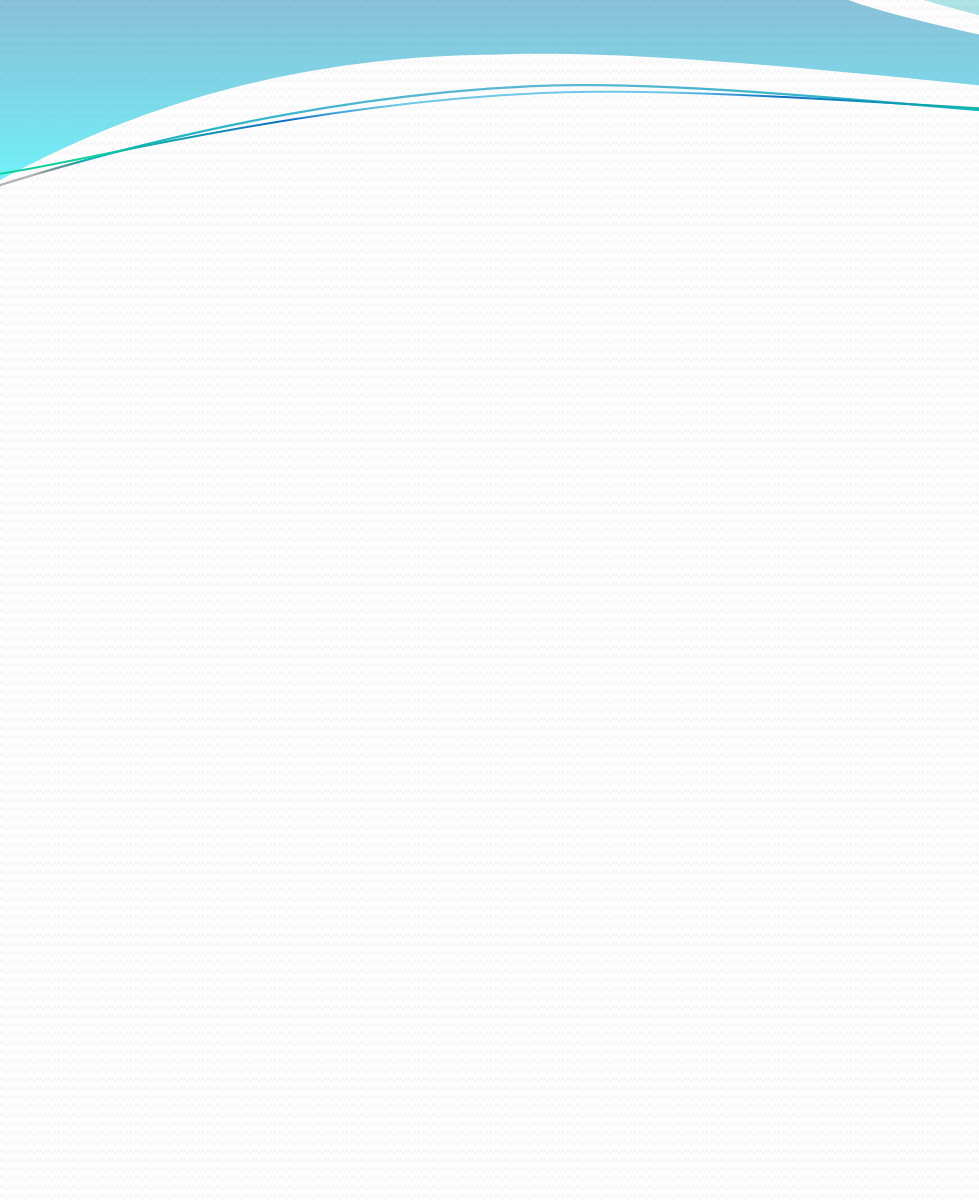


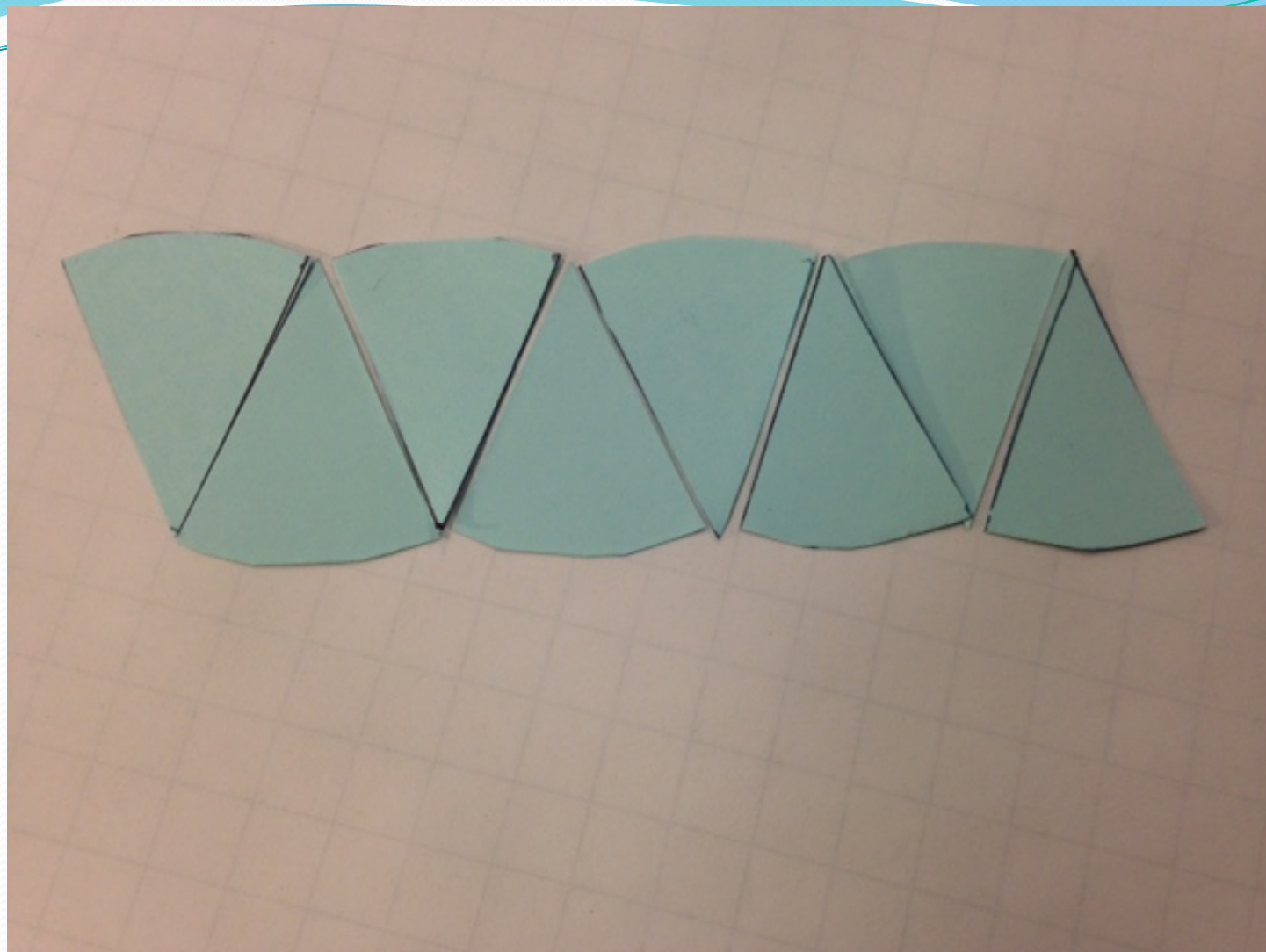
C and D Ratio Table

	Circumference	Diameter	Ratio Value
			
			
			
			

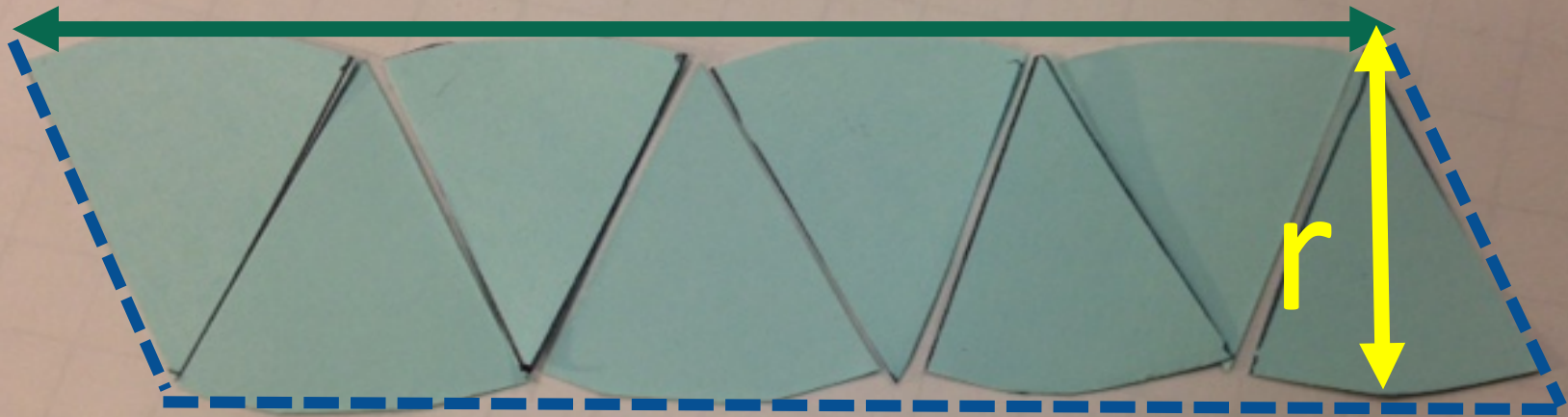
Why Area of Circle = πr^2 ?







$$\frac{1}{2}C$$



$$A = r\left(\frac{1}{2}C\right)$$

$$A = r\left(\frac{1}{2}d\pi\right)$$

$$A = r\left(\frac{1}{2}2r\pi\right)$$

$$A = r(r\pi)$$

$$A = \pi r^2$$

Pythagorean Theorem

*Centre de Ressources
Éducatives du Bas-
Artibonite (CREBA)*
teachers in Haiti
applying (real-life) the
Pythagorean Formula at
a training session in the
City of Saint-Marc.
August 2018



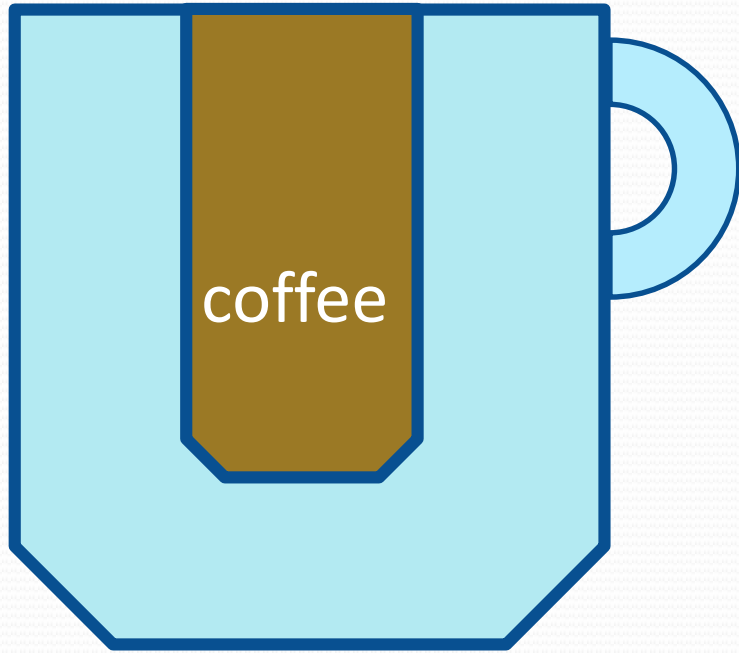
Volume vs. Capacity

What is the difference between

Volume and Capacity?

Volume v. Capacity

- Which cup has the greater volume? Why?
- Which cup has the greater capacity? Why?



Blue Cup



White Cup

Exploring Prism



A rectangular prism can be viewed as is a pile of rectangles. The collection forms a structure having a length, a width, and a height.

Exploring Cylinder

A cylinder can be viewed as a pile of circles. The collection forms a structure having a diameter and a height.



Key Formulae

- $1 \text{ dm}^3 = 1 \text{ kg} = 1 \text{ Liter}$
- $1 \text{ kg} = 2.2 \text{ Lbs.}$
- $1 \text{ cm}^3 = 1 \text{ gram} = 1 \text{ milliliter}$
- $1 \text{ kg} = 2.2 \text{ Lbs.}$
- $F^{\circ} = 1.8C + 32$
- $C^{\circ} = 5/9 (F - 32)$

Capacity

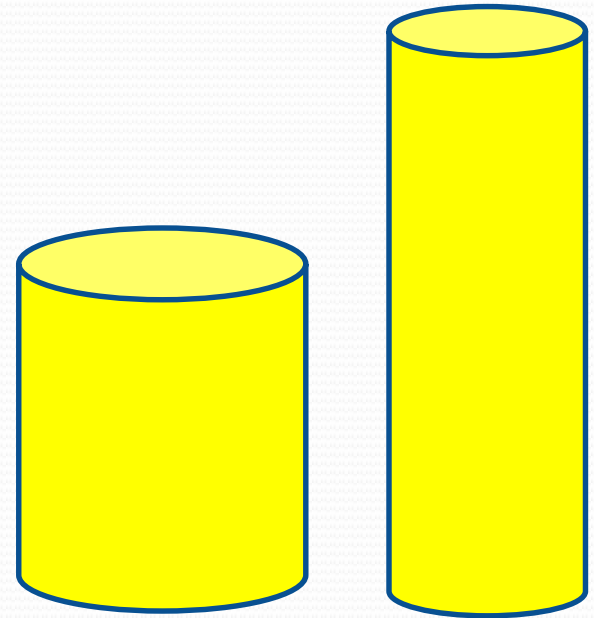
*Centre de Ressources
Éducatives du Bas-
Artibonite (CREBA)*
teachers in Haiti
exploring (real-life) the
notion of capacity of a
cylinder at a training
session in the City of
Saint-Marc and
Gonaïves.
August 2018



Two Cylinders

(Application of the Socratic Method)

Which of the two
cylinders has a greater
capacity? The shorter
one or the taller one?
How do you know?

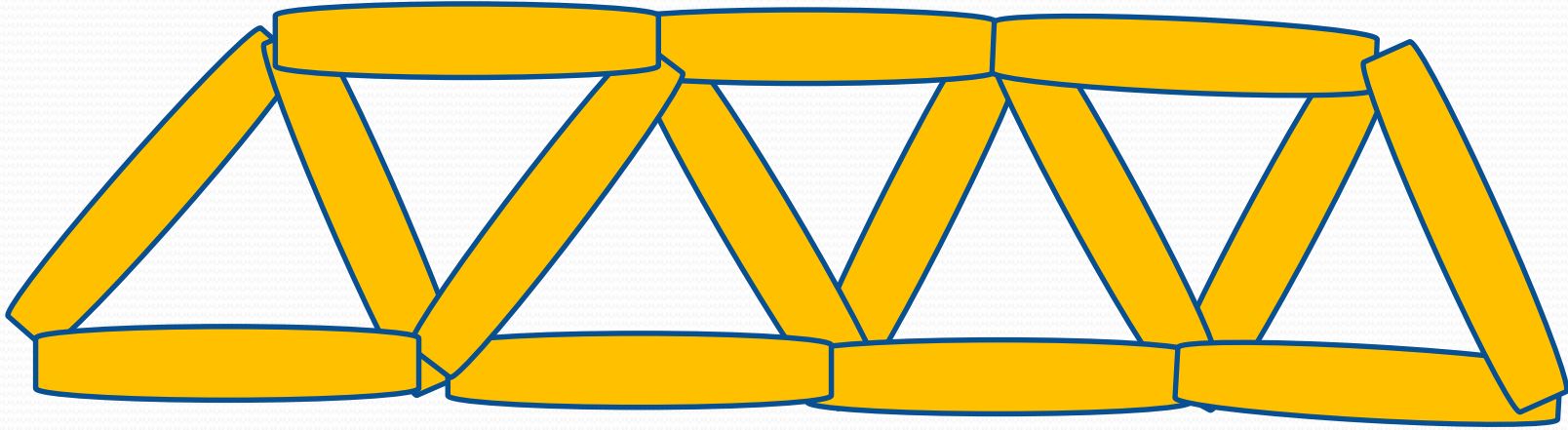




Formulating Equations or Functions

From Patterns

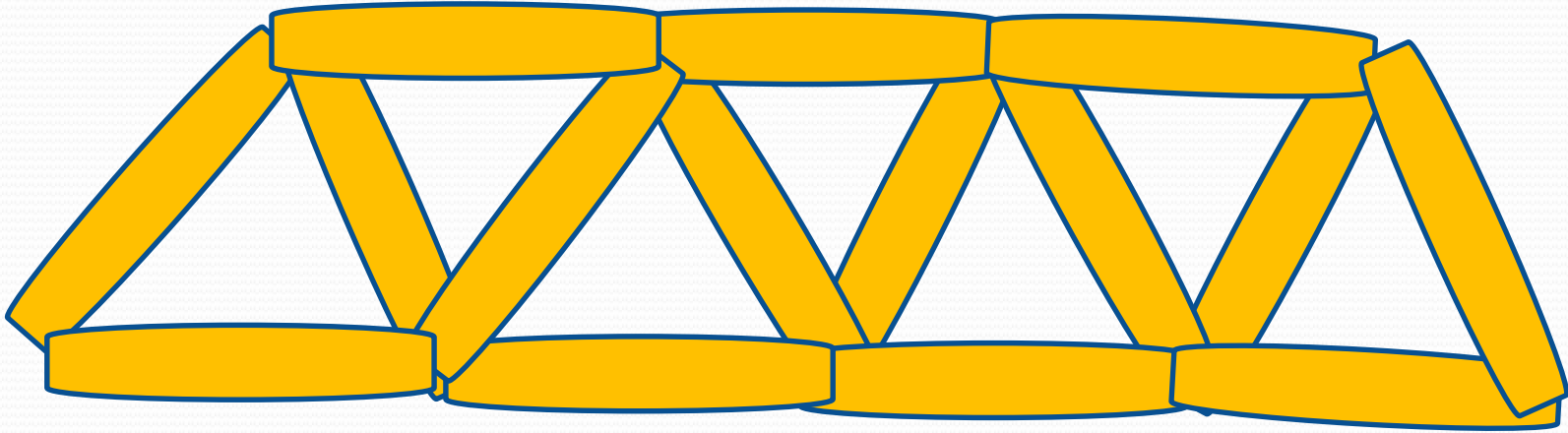
Generating Equations or Functions



Input-Output/Table of Value

t # of Triangles	s # of Sticks
1	3
2	5
3	7
4	9
10	21
1,500	3,001

Formulating Equations or Functions



$$s = 2t + 1$$

or

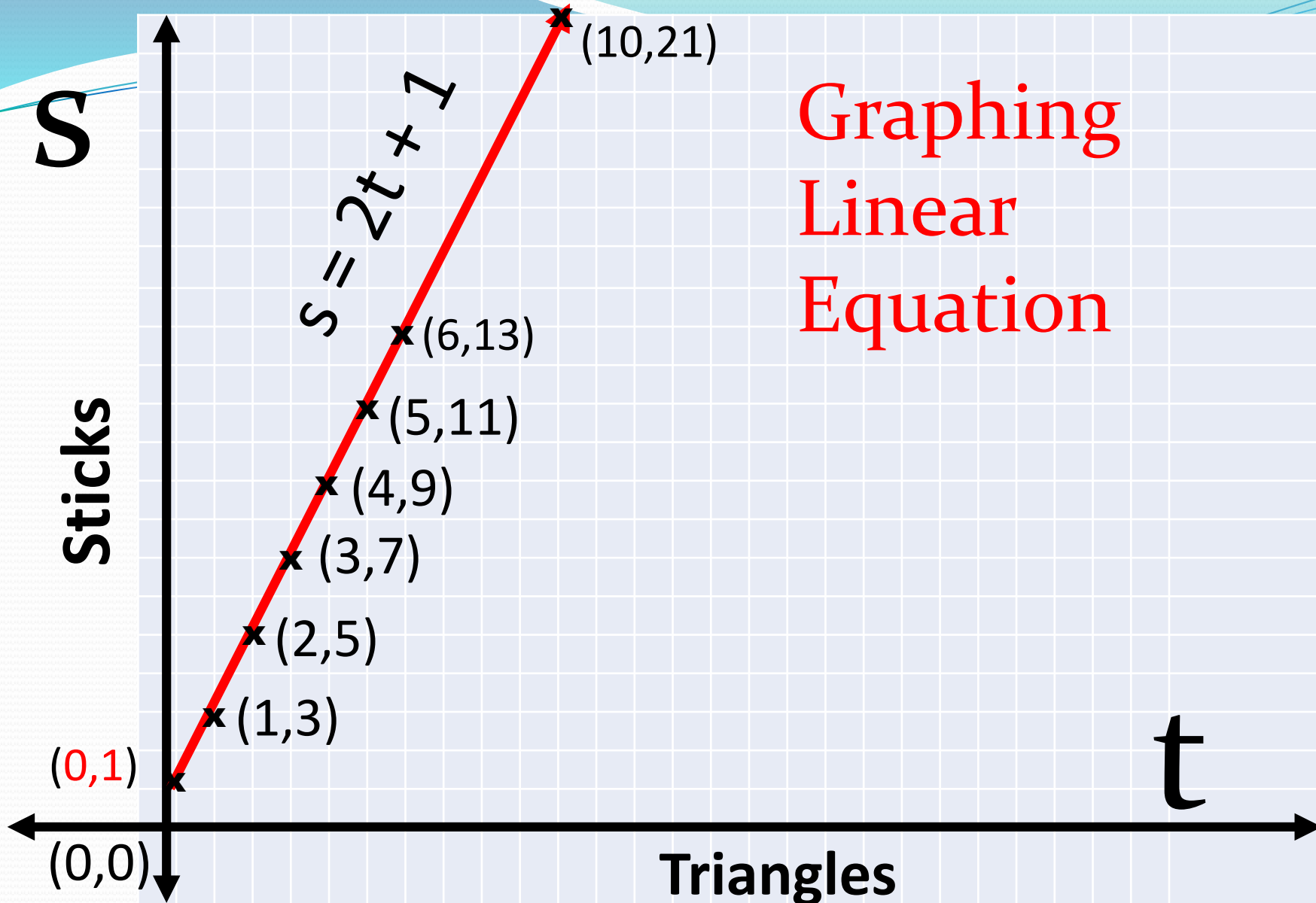
$$f(t) = 2t + 1$$

$$y = 2x + 1$$

or

$$f(x) = 2x + 1$$

Graphing Linear Equation





Equations or Functions with Color Tiles

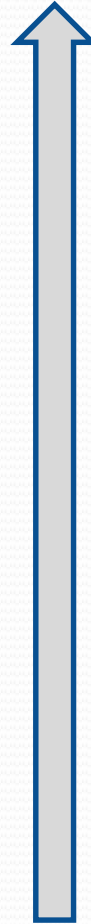
Market 1



Taxi \$6

Market 2

No Taxi



1 for \$0.50



1 for \$2

The Variables or Coordinates



*Independent
Variable x*

*Dependent
Variable y*

Equation / Function

$$y = 0.5x + 6 \quad (1)$$

$$y = 2x \quad (2)$$

x mangoes

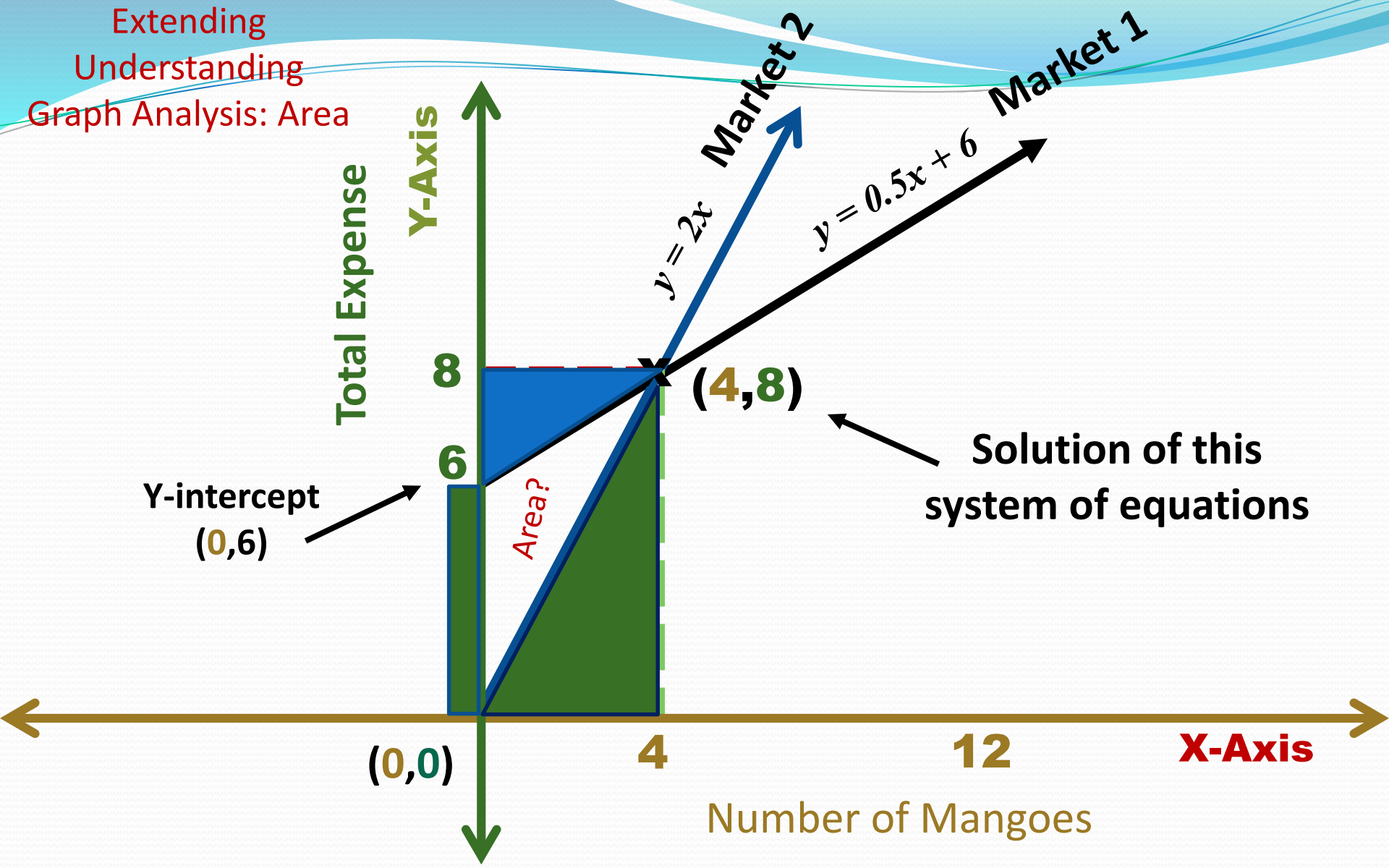
y dollars

Function Machine

Double-Entry Input-output Table

Quantity of Mangoes	MARKET 1 $y = 0.5x + 6$	MARKET 2 $y = 2x$
0	6.00	0.00
1	6.50	2.00
2	7.00	4.00
3	7.50	6.00
4	8.00	8.00
5	8.50	10.00
6	9.00	12.00
7	9.50	14.00
8	10.00	16.00

Extending
Understanding
Graph Analysis: Area



Equations/Functions

Additional Activities

1. The grain of rice story: One grain of rice being doubled every day
2. Stacking up identical books from the floor
3. Stacking up identical books mounted on 6-inch pedestal
4. Cell phone sales competition
5. Mowing the lawn with per-hour pay
6. Shoveling the snow for a \$5-sandwich upfront and per-hour pay
7. Diameter = $2r$
8. Circumference = $3.14d$
9. Area = $3.14r^2$
10. Fahrenheit = $1.8C + 32$

Review: Scaffolding Strategies & Frameworks

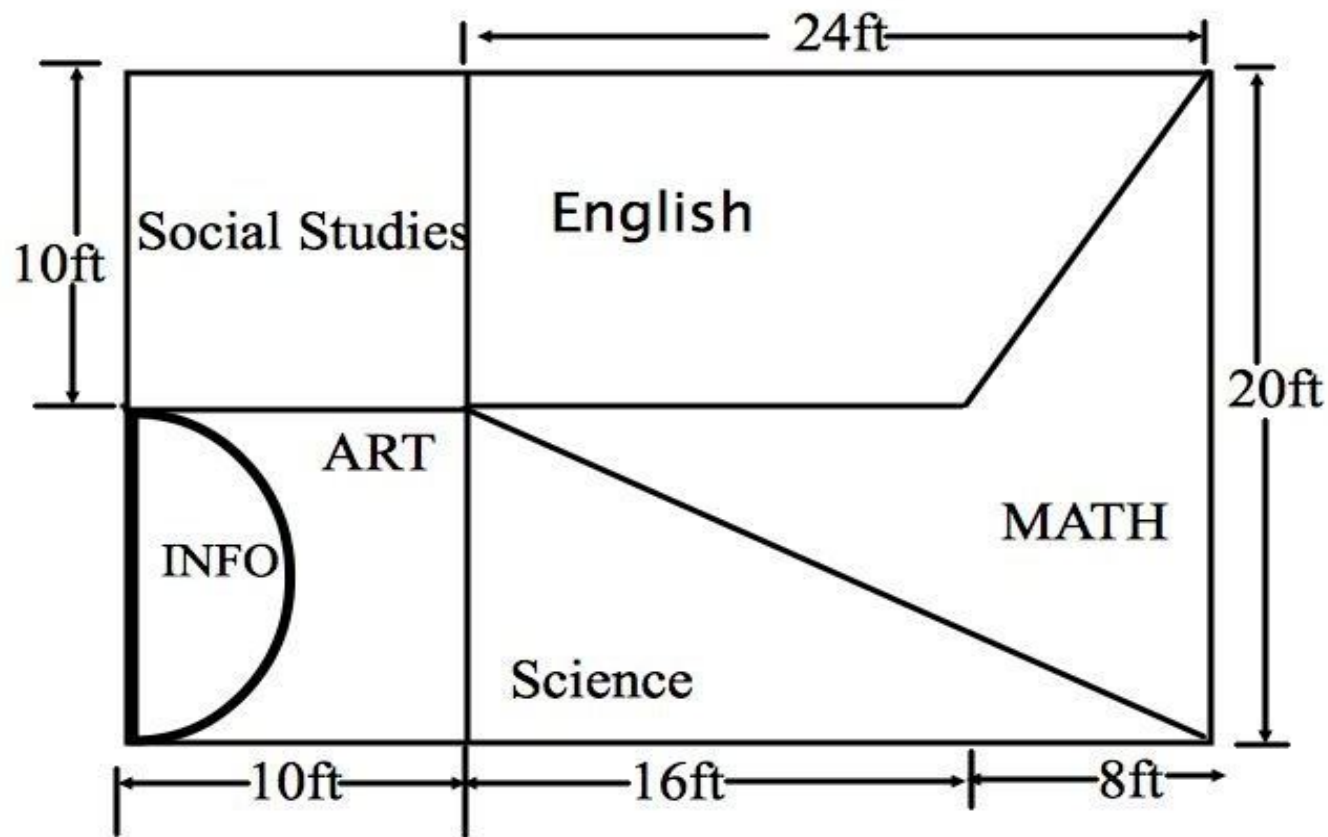
1. Verbal-Visual-Word Association (VVWA)
2. Concept Map
2. Sentence Starters / prompts
3. Word Etymology & Vocabulary
4. KWL Chart
5. Anticipatory Guide
6. Frayer Model
7. Math Glossary Puzzle
8. Flow Chart / Table / Diagram
9. Math Poetry
10. Close Reading
11. Socratic Method for Teaching & Learning
12. Project-based Learning
13. Organic-way Math (under study)
14. Math Games (e.g., ORGABEZ)
15. Etc.



PM Activities

Group Tasks & Presentations

12: 00 – 1:30 PM



New Library Proposal Table

Room	Area	Fraction	Decimal	Percent	Cost
Math					\$400
Info					
English					
Science					
Art					
Social Studies					
Total Library					

A



The 5-gallon Bottles

Same amount of water? When?

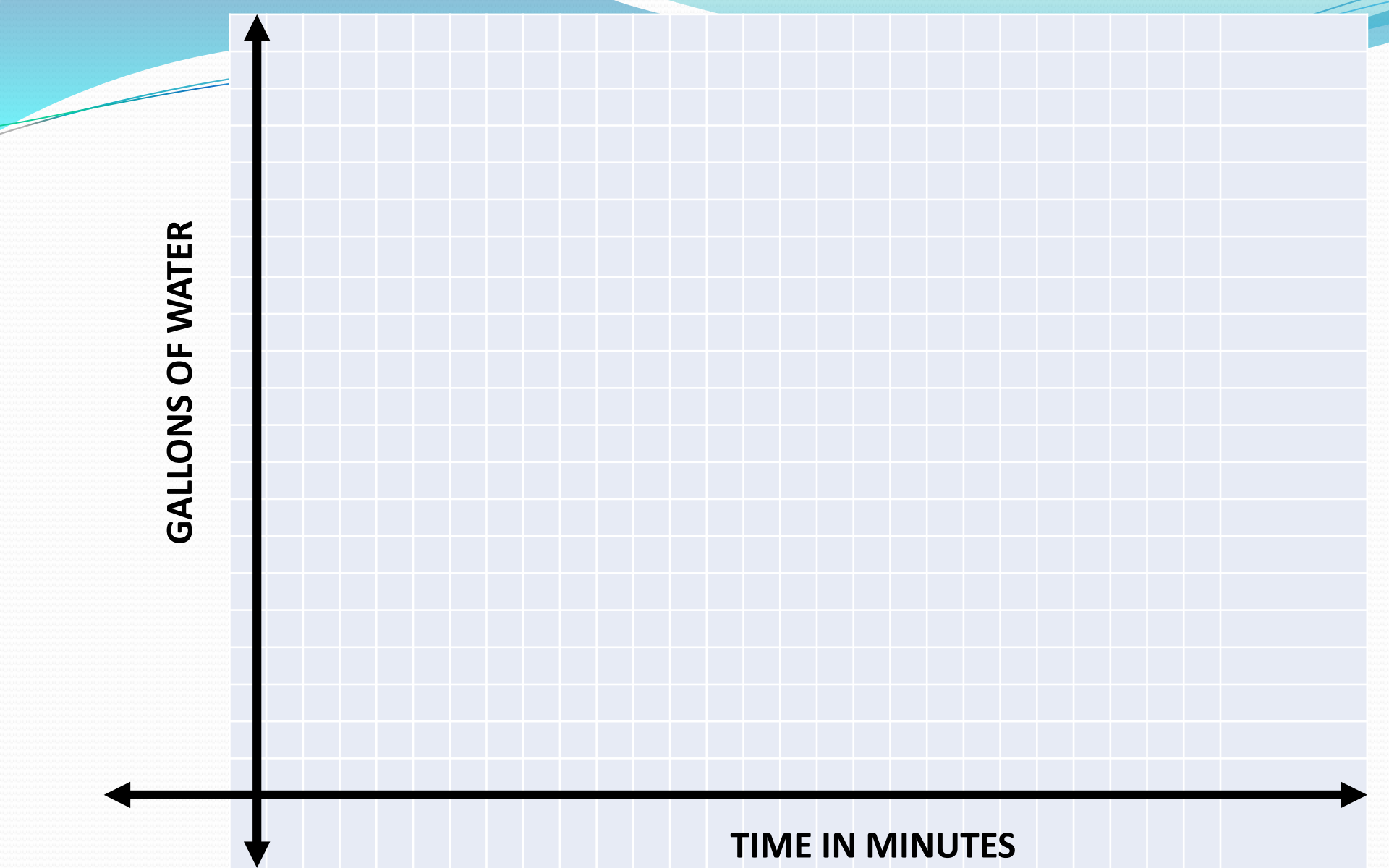
5-gallon water
Bottle to be
emptied in 20
minutes



B



Bottle taking in
 $\frac{1}{4}$ gallon of water
per minute.



GALLONS OF WATER

TIME IN MINUTES

Gas Tank

After traveling a distance of 90 miles, a truck with a full tank has 40 gallons of gas left. The truck continued its journey. After cruising a distance of 360 miles, the vehicle had only 10 gallons of gas left in the tank.

1. How much more (longest) distance could the truck travel before refueling?
2. How much gas fills the tank?

Explain. Use various strategies.



Coordinated Polygons

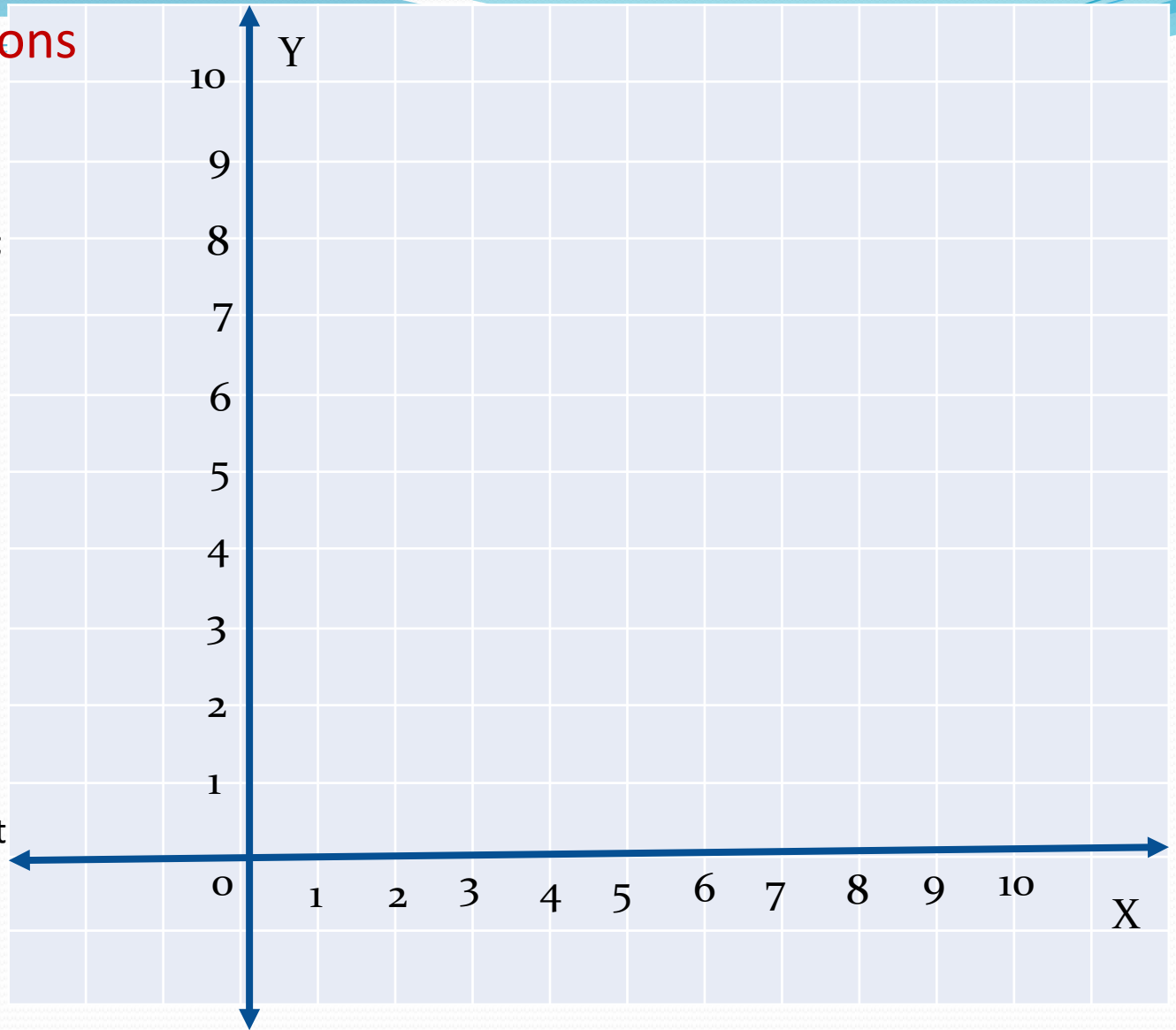
1. Plot the points:

A(3,3); B(5,3); E(4,0); F(2,0);
D(6,0); C(6,6).

2. Connect the points to
create the following
polygons: ABEF, BDE, BCD,
and ABCDF.

3. Find the area of each
polygon and complete the
table.

4. Write your observation
of the table, especially what
amazes you the most.



Coordinated Polygon Table

Polygon	Area	Fraction	Decimal	Percent	Cost
ABEF					\$500
BDE					
BCD					
ABCDF					

Observation:-----

The Better Buy

Use cm ruler to investigate which of the two containers filled with the same brand of substance is the better buy. Prove it theoretically (formula) and empirically (with rice).



Mango Transaction

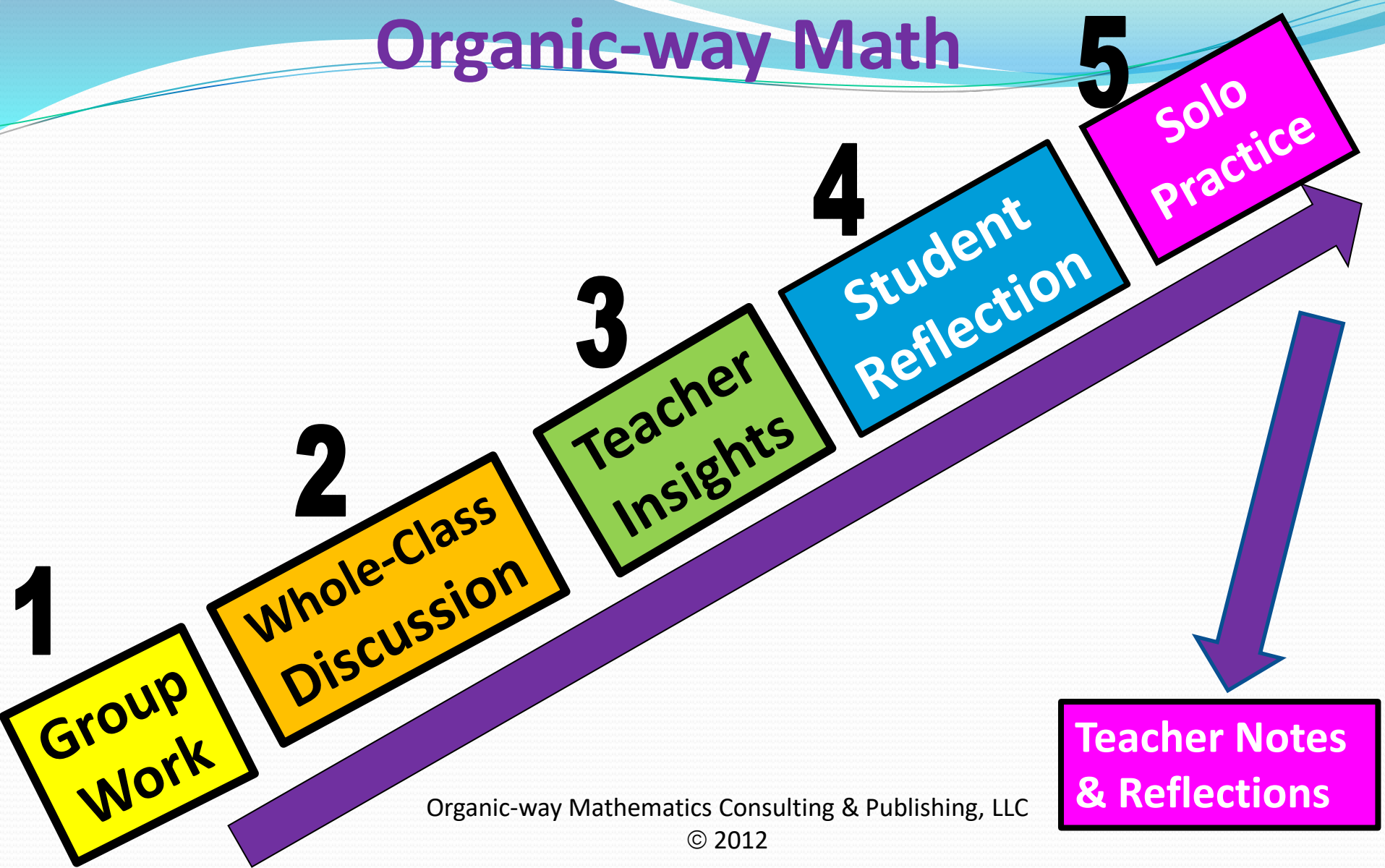
Universal Design For Learning

1. Multiple forms of representation
2. Multiple forms of engagement
3. Multiple forms of expression

Some Research-based 21st Century Models

- Standards-based Model
- Constructivist Model
- Discovery Model
- Inquiry-based Model
- Project-based Model
- Japanese Model (Neriage)
- Chinese Model (Bantu)
- Organic-way Model (under study)
- Etc.

Organic-way Math



New York State Language RBERN @ NYU

726 Broadway, 5th Floor

New York, NY 10003



Thank You

References

Organic-way Mathematics Consulting & Publishing, LLC.

<https://global.indiana.edu/documents/global-perspectives/clarifying-and-probing-questions-handout-step-2-define.pdf>